Nuclear $\beta$ Decay

- Two types of $\beta$ decay, $\beta^-$ (electron) and $\beta^+$ (positron).
- Nuclear $\beta^-$ decay occurs when a neutron decays into a proton, electron and anti-neutrino.
- Mediated by the weak nuclear force.
In the Standard Model, the $\beta$ decay Hamiltonian has the $V – A$ form

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left[ \bar{e} \gamma_\mu (1 - \gamma_5) \nu_e \bar{u} \gamma^\mu (1 - \gamma_5) d \right] + \text{H.c.}$$
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The most general form of the effective Hamiltonian describing $n \to pe^- \bar{\nu}_e$ ($\beta^-$ decay) is

$$\mathcal{H}_\beta \simeq \mathcal{H}_{V,A} + \mathcal{H}_S + \mathcal{H}_T,$$

where $\mathcal{H}_{V,A}$ is the vector and axial-vector term, $\mathcal{H}_S$ is a scalar contribution term, and $\mathcal{H}_T$ is a tensor contribution term.
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Scalar Currents

The Standard Model of particle physics is an incomplete theory, thus we look to extensions of the SM. In particular, to set limits on the existence of fundamental or induced scalar interactions we turn to the $ft$ values for superallowed Fermi $\beta$ decays.
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**Definition**

Superallowed Fermi $\beta$ decays are beta decays between isobaric analogue states (ie. $T_i = T_f$) where the parent and daughter nuclei have $J^\pi = 0^+$. 
Why \( f t \) Values?

- Have confirmed the CVC hypothesis at the level of \( 1.3 \times 10^{-4} \)
- Provide the most precise value for \( V_{ud} \) to date
- After making theoretical QCD and QED corrections, corrected \( f t \) values, denoted \( F_t \), are expected to be nucleus independent
- Set limits on the existence of a fundamental or induced scalar interaction in the Standard Model
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To place further constraints on possible extensions of the Standard Model:

$$f t \text{ value precision } \leq 0.1\% \rightarrow \beta \text{ decay half-life precision } \leq 0.05\%.$$
### Corrected $t_f$ values

<table>
<thead>
<tr>
<th>Element</th>
<th>$t_f$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10}$C</td>
<td>3090 ± 74</td>
</tr>
<tr>
<td>$^{14}$O</td>
<td>3085 ± 62</td>
</tr>
<tr>
<td>$^{26}$Al$^m$</td>
<td>3080 ± 54</td>
</tr>
<tr>
<td>$^{34}$Ar</td>
<td>3075 ± 50</td>
</tr>
<tr>
<td>$^{34}$Cl</td>
<td>3070 ± 46</td>
</tr>
<tr>
<td>$^{38}$K$^m$</td>
<td>3065 ± 42</td>
</tr>
<tr>
<td>$^{42}$Sc</td>
<td>3060 ± 38</td>
</tr>
<tr>
<td>$^{50}$Mn</td>
<td>3055 ± 34</td>
</tr>
<tr>
<td>$^{54}$Co</td>
<td>3050 ± 34</td>
</tr>
<tr>
<td>$^{74}$Rb</td>
<td>3045 ± 26</td>
</tr>
<tr>
<td>$^{14}$O</td>
<td>3040 ± 22</td>
</tr>
<tr>
<td>$^{10}$C</td>
<td>3035 ± 14</td>
</tr>
<tr>
<td>$^{14}$O</td>
<td>3030 ± 10</td>
</tr>
<tr>
<td>$^{10}$C</td>
<td>3025 ± 5</td>
</tr>
</tbody>
</table>

**Note:**
- $t_f$ values are corrected for half-life measurement of $^{14}$O.
- The graph shows the relationship between the atomic number ($Z$) of the daughter and the corrected $t_f$ values for various elements.

---

**Figure:**
- The graph is labeled with the atomic numbers of the daughter elements.
- Error bars indicate the uncertainty in the measurements.
Corrected $f_t$ values

$$\frac{C_S}{C_V} = 0.002$$
How do we measure $ft$ Values?

In order to measure $ft$ values, we must measure:

- $Q$-value, the total transition energy
- $T_{1/2}$, the half-life of the parent
- $\beta$ branching ratios
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- $Q$-value, the total transition energy
- $T_{1/2}$, the half-life of the parent
- $\beta$ branching ratios

If the primary $\beta$ branch emits a characteristic $\gamma$-ray we may measure the half-life via:

- direct $\beta$ counting
- $\gamma$ photopeak counting
One of the most precisely measured superallowed half-lives known is \(^{14}\text{O}\). An unsettling systematic effect arises when comparing the results from the two experimental methods.

\[
\begin{align*}
T_{1/2}(\gamma) &= 70.616(13) \text{ s} \\
T_{1/2}(\beta) &= 70.696(52) \text{ s}
\end{align*}
\]

These discrepancies provide the motivation for a simultaneous direct \(\beta\) and \(\gamma\)-ray counting experiment.

Similarly, a systematic bias occurs with the \(^{10}\text{C}\) half-life where the precise \(\beta\) counting experiment disagrees at a level of \(3\sigma\), or \(0.10\%\) with the \(\gamma\) counting method.
Half-life Measurement of $^{14}$O

**Introduction**

**Motivation**

- **Half-life of $^{14}$O:**
  - $T_{1/2}^{\gamma}(^{14}$O) = 70.616(13) s
  - $T_{1/2}^{\beta}(^{14}$O) = 70.696(52) s

- **Half-life of $^{10}$C:**
  - $T_{1/2}^{\gamma}(^{10}$C) = 19.290(12) s
  - $T_{1/2}^{\beta}(^{10}$C) = 19.310(4) s
A strong superallowed program is in place at TRIUMF’s Isotope Separator and Accelerator (ISAC) facility, where the primary driver is a 500 MeV cyclotron which provides intense beams of up to 100 \( \mu \)A of protons to thick layered-foil targets which produce radioisotopes through spallation.
Proposed Experiment

- A simultaneous $\gamma$ and $\beta$ counting experiment for $^{14}$O was run at TRIUMF in November 2011.
- The $8\pi$ facility was used to make the measurements.
- A new detector set-up, including the $8\pi$ Gamma-Ray Spectrometer, Scintillating Electron-Positron Tagging Array (SCEPTAR), and Zero-Degree Scintillator (ZDS), is in place and it is being investigated.
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- In follow-up experiments, the General Purpose Station (GPS) will be used for both $^{10}$C and $^{14}$O measurements.
- The half-life of $^{10}$C will also be measured at $8\pi$ in a simultaneous $\beta$-$\gamma$ experiment.
8π Spectrometer

- Spherical array of 20 Compton-suppressed HPGe detectors
- Covers approximately 13% of the 4π solid angle
- Detects γ-rays emitted from excited daughter states
Pile-up corrections for high-precision superallowed $\beta$ decay half-life measurements via $\gamma$-ray photopeak counting


Departments:

1. Department of Physics, University of Guelph, Guelph, Ont., Canada N1G 2W1
2. TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, Canada V6T 2A3
3. Department of Astronomy and Physics, St. Mary's University, Halifax, NS, Canada B3H 3C3
4. School of Physics, Georgia Institute of Technology, Atlanta, GA 30332 0490, USA
5. Department of Physics, Queen's University, Kingston, Ont., Canada K7L 3N6
6. Department of Physics, Colorado School of Mines, Golden, CO 80401, USA
7. Department of Physics and Astronomy, McMaster University, Hamilton, Ont., Canada L8S 4K1
8. Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803 4001, USA

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Abstract

A general technique that corrects $\gamma$-ray gated $\beta$ decay-curve data for detector pulse pile-up is presented. The method includes corrections for non-zero time-resolution and energy-threshold effects in addition to a special treatment of saturating events due to cosmic rays. This technique is verified through a Monte Carlo simulation and experimental data using radioactive beams of $^{26}$Na implanted at the center of the 8$\pi$ $\gamma$-ray spectrometer at the ISAC facility at TRIUMF in Vancouver, Canada. The $\beta$-decay half-life of $^{26}$Na obtained from counting 1809-keV $\gamma$-ray photopeaks emitted by the daughter $^{26}$Mg was determined to be $T_{1/2} = 1.07167 \pm 0.00055$ s following a 27$\pi$ correction for detector pulse pile-up. This result is in excellent agreement with the result of a previous measurement that employed direct $\beta$ counting and demonstrates the feasibility of high-precision $\beta$-decay half-life measurements through the use of high-purity germanium $\gamma$-ray detectors. The technique presented here, while motivated by superallowed-Fermi $\beta$ decay studies, is general and can be used for all half-life determinations (e.g. $\alpha$, $\beta$, X-ray, fission) in which a $\gamma$-ray photopeak is used to select the decays of a particular isotope.

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**SCEPTAR**

- Spherical array of 20 thin plastic scintillating $\beta$ detectors (10 per hemisphere) surrounding the implantation point of the radioactive ion beam inside the central vacuum chamber of $8\pi$

- Each scintillator sits in front of a HPGe detector to provide $\beta-\gamma$ coincidence information
Zero-Degree Scintillator

- Fast plastic scintillator behind implantation site, replacing the back half of SCEPTAR
- Detects $\beta$ particles directly
- Beam is implanted onto tape, data is recorded, tape is moved once nucleus of interest has decayed
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- Detects $\beta$ particles directly
- Beam is implanted onto tape, data is recorded, tape is moved once nucleus of interest has decayed
- It has never been used for high-precision half-life measurements
Experiment Overview

- Experiment run in November 2011 using the $8\pi$, SCEPTAR & ZDS
- 95 runs were performed where each run consisted of:
  1 min background — 3 min beam on — 23 min decay
- Beam of $^{12}\text{C}^{14}\text{O}$ with $^{26}\text{Na}$ contaminant
- Various settings such as deadtime and shaping time were varied run-by-run to investigate systematics
Deadtime Corrections

- Five multichannel scaler modules were used to independently record the ZDS decay data.
- Fixed, nonextendable deadtimes (chosen to be longer than the series deadtimes of the system) were applied to each MCS.
- The deadtimes were measured via the source-plus-pulser method to be 1.981(3) µs, 5.002(4) µs, 10.001(4) µs, 20.006(7) µs, and 29.991(9) µs.
- To correct the data for the deadtime effects, the following equation was used:

\[ y_i = \frac{n_i}{1 - n_i \left( \frac{\tau}{t_b} \right)} \]
The data was then fit with a two exponential decays, a contaminant of $^{26}\text{Na}$ (with a half-life fixed at its central value of 1.07128 s) and the $^{14}\text{O}$, plus a constant background. The fit function, of four free parameters, can be expressed as:

$$y_{\text{fit}}(t) = \int_{t_i}^{t_f} a_1 \exp \left( -\frac{\ln 2 \cdot t}{a_2} \right) + a_3 \exp \left( -\frac{\ln 2 \cdot t}{a_4} \right) + a_5 \, dt$$

The level of contamination of the $^{26}\text{Na}$ was relatively large ($\gtrsim 10\%$), but by waiting several seconds after the beam turned off most of the sodium decayed leaving a relatively pure ($\geq 99.9\%$) sample of $^{14}\text{O}$. 
PRELIMINARY Sample Fit

**All Runs (summed)**

MCS24 (2 µs deadtime)

![Graph showing a decay curve with a fitted line and residual plot.]

\[ T_{1/2}^{(14}\text{O}) = 70.580(20) \text{ s}, \chi^2/\nu = 1.17 \]

Accepted \( T_{1/2} = 70.620(15) \text{ s} \)
$^{14}$O Half-life
MCS24 (2 µs deadtime)

Average $T_{1/2}^{^{14}\text{O}} = 70.572(19)$ s, $\chi^2/\nu = 1.11$

Accepted $T_{1/2} = 70.620(15)$ s
The feasibility of the Zero Degree scintillator for high-precision half-life measurements is being investigated.

The analysis is still in preliminary stages and more in-depth work must be done in the coming months.

We are preparing for the rerunning of this experiment in Fall/Winter and the $^{10}$C superallowed Fermi $\beta$ decay experiments in the future.

After obtaining high statistics experiments at $8\pi$ and GPS we will be able to address the current systematic bias existing from experimental method used.

These experiments will help test the limits of induced and fundamental scalar interactions and extensions of the Standard Model.
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Determining the $f t$ Values

From Fermi’s Golden Rule, we have that

$$ft = \frac{K}{|M_{f,i}|^2 G_v^2}$$

Assuming isospin is a perfect symmetry, $|M_{f,i}|^2$ for $\beta^{\pm}$ decay from $0^+ \rightarrow 0^+$ states is the expectation value for the isospin lowering (raising) operator.

$$|M_{f,i}|^2 = \langle T, T_3 \mp 1|\hat{T}^{\mp}T, T_3\rangle^2$$

$$= (T \pm T_3)(T \mp T_3 + 1)$$

Specifically, both $^{10}\text{C}$ and $^{14}\text{O}$ are $T = 1$, $T_z = -1$ $\beta^+$ emitters. Thus, we clearly see

$$|M_{f,i}|^2 = (1 + 1)(1 - 1 + 1) = 2.$$
The phase space integral, $f$, is defined as

$$f = \int_{1}^{W_0} p \ W \ (W_0 - W)^2 \ F(Z, W) \ S(Z, W) \ dW,$$

where $W$ is the electron total energy in electron rest-mass units, $W_0$ is the maximum value of $W$, $p$ is the electron momentum, $Z$ is the charge number of the daughter nucleus, $F(Z, W)$ is the Fermi function, and $S(Z, W)$ is the shape-correction factor.

The partial half-life, $t$, is defined as

$$t = \frac{\ln 2}{\lambda_{i \rightarrow f}} = \frac{T_{1/2}}{B_f}.$$

Combining all of this we have that

$$ft = \frac{2\pi^3 \hbar^7 \ln 2}{|M_{f,i}|^2 \ G_V^2 m_e^5 c^4}$$
To measure the deadtime ($\tau$) of a system we use two sources, $A$ and $B$, that we count independently and in combined form $C$. Generally, we use artificial periodic pulses (of frequency $n_p^0$) for one of the random sources and a random source of rate $n_r$ when counted alone.

Once combined, the recording rate for periodic pulses is

$$n_p = n_p^0(1 - n_r\tau),$$

while the random rate is

$$n'_r = n_r(1 - n_p\tau) = n_r[1 - n_p^0(1 - n_r\tau)\tau].$$

The total combined counting rate, $n_{rp}$, is just the sum of the $n_p$ and $n'_r$

$$n_{rp} = n_p^0(1 - n_r\tau) + n_r[1 - n_p^0(1 - n_r\tau)\tau]$$

$$= n_p^0 + n_r - 2n_p^0n_r\tau + n_r^2n_p^0\tau^2,$$

or

$$\tau = \frac{n_p^0 + n_r - n_{rp}}{2n_p^0n_r(1 - n_r\tau/2)},$$

which can be solved by iteration.
Deadtime Effects

![Graph showing deadtime effects with a curve for deadtime corrected data and another for raw data. The x-axis represents bins (0.1 s/bin), and the y-axis represents counts. The graph illustrates the decline of counts over time with and without deadtime correction.]
Chop Plot

MCS24 (2 µs deadtime)

# Channels Chopped (6 s/channel)

$^{14}\text{O}$ Half-life (s)

Alex Laffoley (University of Guelph)

February 24, 2012