Quarkonium Formation Time in Heavy Ion Collisions

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- Medium effects on quarkonium production
- Formation time effect on quarkonium production
- Quarkonium formation time in vacuum
- Quarkonium formation time in QGP
- Quarkonium formation time in heavy ion collisions

Based on work with Taesoo Song and Su Houng Lee: PRC 87, 034910 (2013); 91, 044909 (2015).
Medium effects on quarkonium production

- Cold nuclear matter effects
  - Cronin effect: Gluon-nucleon scattering before quarkonium production from gluon-gluon fusion
  - Shadowing effect: Medium modification of gluon distribution function in nuclei
  - Nuclear absorption by passing nucleons

- Hot medium effects
  - Color screening: Melting of quarkonia (Matsu & Satz)
  - Thermal dissociation: Broadened width (Rapp …)
  - Regeneration: Enhanced production (Thews, Braun-Munzinger)
  - Relaxation effect: Reduced regeneration rate (Rapp, …)

- Both cold nuclear absorption and hot medium effects are affected by quarkonium formation time (Gunion & Vogt, ….)
### Nuclear modification factor for J/ψ

**Song, Han & Ko, PRC 84, 034907 (2011)**

- **Potential**: Screened Cornell potential
- **Dissociation**: NLO perturbative QCD with massive thermal partons
- **Dynamics**: 2+1 ideal hydro
- **Formation time as parameter**

- Most J/ψ are survivors from initially produced
- Kink in $R_{AA}$ is due to the onset of initial temperature above the J/ψ dissociation temperature in QGP
- Inclusion of shadowing reduces slightly $R_{AA}$
Nuclear modification factor for $\Upsilon(1S)$

Song, Han & Ko, PRC 85, 014902 (2012)

- Regeneration contribution is negligible
- Primordial excited bottomonia are largely dissociated
- Medium effects on bottomonia reduce $R_{AA}$ of $\Upsilon(1S)$
Y(1S) nuclear modification factor at LHC from theoretical models

1) Strickland, PRL 107, 132301 (2011)

2) Zhuang et al.,

3) Emerick, Zhao & Rapp, EPJA 48, 72 (2012)

- Potential: in-medium Cornell
- Disso.: LO pQCD
- Dynamics: anisotropic hydro

- Potential: U or F
- Disso.: vacuum LO gluo-disso.
- Dynamics: ideal hydro

4) Brezininki & Wolschin, PLB 707, 534 (2012): schematic estimate using in-medium LO gluo-dissociation
Thermal decay width of Y(1S) in different models

- Thermal decay width
  - Rapp: quasielastic scattering
  - Zhuang: OPE by Peskin
  - Strickland: LO pQCD
  - Song: NLO pQCD with massive thermal partons

- Very different models are used for calculating thermal decay widths are used
Formation time effect on quarkonium production

Ganesh and Mishira, PRC 91, 034901 (2015)

\[ \tau_f(T) \approx \tau(0) \frac{E_{\text{bind}}(0)}{E_{\text{bind}}(T)} \]

Binding energy \( E_{\text{bind}}(T) \) calculated using screened Cornell-like heavy quark potential [Nendzig & Wolschin, PRC 87, 024911 (2013)]

Vacuum formation time \( \tau(0) \); 0.76 fm (\( \Upsilon(1S) \)), 1.9 fm (\( \Upsilon(2S) \)) [Gunion & Vogt NPB 492, 301 (1997)]

FIG. 7. (Color online) CMS data [19] compared with simulation results with only color screening for \( \Upsilon(1S) \).

FIG. 8. (Color online) CMS data [19] compared with simulation results with only color screening for \( \Upsilon(2S) \).
**Quarkonium formation time in vacuum**  
Kharzeev and Thews, PRC 60, 041901 (1997)

- **Space-time correlator of heavy quark currents**  
  \[ J_\mu(x) = \bar{Q}(x) \gamma_\mu Q(x) \]

  \[
  \Pi_{\mu\nu}(x) = \langle 0 | T \{ J_\mu(x) J_\nu(0) \} | 0 \rangle = \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi(q^2)
  \]

  Using dispersion relation  
  \[ \Pi(q^2) = \frac{1}{\pi} \int ds \frac{\text{Im} \Pi(s)}{s-q^2} \]

  \[
  \Pi(x) = \Pi_{\mu}^\mu(x) = \frac{3}{\pi} \int ds \text{Im} \Pi(s) \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} = \frac{3}{\pi} \int ds \text{Im} \Pi(s) D(s, x^2)
  \]

  Causal propagator in Euclidean coordinate space  
  \[ D(s, \tau^2 = -x^2) = \frac{\sqrt{s}}{4\pi^2 \tau} K_1(\sqrt{s\tau}) \]

  Imaginary part of heavy quark pair polarization function

  \[
  \text{Im} \Pi(s) = \frac{s}{(4\pi\alpha)^2} \sigma(e^+e^- \rightarrow Q\bar{Q}, s) = \frac{1}{12\pi} \frac{\sigma(e^+e^- \rightarrow Q\bar{Q}, s)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-, s)} \equiv \frac{1}{12\pi} R(s)
  \]

- **Correlator** is a sum of propagators of physical states weighted by their production probabilities in a hard process.
Cross section ratio in terms of resonance and continuum contributions

\[ R(s) = \frac{\sigma(e^+e^-\rightarrow Q\bar{Q},s)}{\sigma(e^+e^-\rightarrow \mu^+\mu^-,s)} = \sum_i R_i(s) + R_{\text{cont}}(s) \]

\[ R_i(s) = \frac{72\pi e_Q^2}{M_i^2} |\psi_i(0)|^2 \frac{\Gamma_i/2}{(\sqrt{s} - M_i)^2 + \Gamma_i^2/4} \]

\[ R_{\text{cont}}(s) = 3e_Q^2 \theta(s - s_{\text{th}}) \]

Heavy quark vector current correlator \( \Pi(\tau) = \sum_i \Pi_i(\tau) + \Pi_{\text{cont}}(\tau) \)

\[ \Pi_i(\tau) = \frac{9e_Q^2 M_i^2}{\pi^2 \tau} |\psi_i(0)|^2 K_1(M_i \tau) \]

\[ \Pi_{\text{cont}}(\tau) = \frac{3e_Q^2}{8\pi^4 \tau^6} \int_{\sqrt{s_{\text{th}}}}^{\infty} dx x^4 K_1(x) \]

\( M_i \): given by measured quarkonium mass
\( \psi_i(0) \): from quarkonium decay width
\( \Gamma_i^{e^+e^-} = \frac{16\pi \alpha^2 e_Q^2}{M_i^2} |\psi(r = 0)|^2 \)
\( \sqrt{s_{\text{th}}} \): taken to be twice heavy quark mass
Since \( \Pi_i(\tau \to 0) \sim \frac{1}{\tau^2}, \quad \Pi_i(\tau \to \infty) \sim \frac{1}{\tau^{3/2}} e^{-M_i \tau} \)

\( \Pi_{\text{cont}}(\tau \to 0) \sim \frac{1}{\tau^6}, \quad \Pi_{\text{cont}}(\tau \to \infty) \sim \frac{1}{\tau^{5/2}} e^{-2\sqrt{s_{\text{th}} \tau}} \)

correlator is dominated by continuum initially and by the ground state at later time

If \( \text{Im} \Pi(s) \approx \delta(s-m_1^2) + \delta(s-m_2^2) \), then

\[
\Pi(\tau) \approx \tau^{-3/2} e^{-m_1 \tau} + \tau^{-3/2} e^{-m_2 \tau} \\
= \tau^{-3/2} e^{-m_1 \tau} [1 + e^{-(m_2-m_1) \tau}]
\]

Since the correlator approaches the form \( \Pi(x) \approx \tau^{3/2} e^{-m_\tau} \) after the time \( 1/(m_2-m_1) \), the formation time can be estimated as \( \tau_f \approx \frac{1}{m_2-m_1} \)

More accurately, consider distribution of formation times \( P(\tau) = dF(\tau)/d\tau \) in terms of derivative of the fraction of ground state in the correlator \( F(\tau) = \Pi_0(\tau)/\Pi(\tau) \)

\[
\tau_f = \langle \tau \rangle = \frac{\int d\tau P(\tau) \tau}{\int d\tau P(\tau)}
\]
Kharzeev and Thews, PRC 60, 041901 (1997)

\[ \tau_f(J/\psi) = 0.44 \text{ fm}/c, \quad \tau_f(\Upsilon) = 0.32 \text{ fm}/c \]
**Quarkonium formation time in QGP**

Need quarkonium mass, wave function at origin and continuum threshold in QGP, which can be determined if the heavy quark potential in QGP is known

- Free energy $F$ of a heavy quark pair from LQCD [Kacmareck, EJP 61, 811 (2009)]

  Two limits of the potential:

  $$V(r, T) = F$$
  or $$V(r, T) = U = F + TS$$

  Schroedinger equation at finite T:

  - binding energy $\varepsilon(T)$ and mass
  - radius $R(T)$ and wave function at origin $\psi(0)$
  - continuum threshold

  $$\sqrt{S_{th}} = 2m_Q + V(r = \infty)$$

Dissociation temperature:

$$\varepsilon(T_D) \to 0, R(T_D) \to \infty$$
- **J/ψ mass and wave function at origin at finite temperature**

\[
\left[-\frac{\nabla^2}{m_c} + \tilde{V}(r, T)\right] \psi(r, T) = -\epsilon \psi(r, T)
\]

\[
\tilde{V}(r, T) \equiv V(r, T) - V(r = \infty, T) \to 0 \text{ as } r \to \infty
\]

\[
\epsilon = 2m_c + V(r \to \infty, T) - M
\]

Results using lattice free energy as heavy quark potential agree with those from
QCD sum rule analysis of heavy quark vector current correlator using gluon
condensates from lattice QCD [Morita and Lee, PRL 100, 022301 (2008), …..]
- Upsilon wave function in QGP

Wave function at origin also decreases with temperature for both \( \Upsilon(1S) \) and \( \Upsilon(2s) \)
Distribution of formation time $P(\tau) = \frac{dF(\tau)}{d\tau}$ for $Y(1S)$

Distribution of formation time shifts to longer time as temperature increases, resulting in a longer average formation time
Formation times of quarkonia increase with increasing temperature and are larger for excited states.
Quarkonium formation time in Au+Au collisions at 200 GeV

QGP evolution described by 2+1 ideal hydro

<table>
<thead>
<tr>
<th>States</th>
<th>$J/\psi$</th>
<th>$\psi'$</th>
<th>$\Upsilon(1s)$</th>
<th>$\Upsilon(2S)$</th>
<th>$\Upsilon(3S')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formation time (fm/c)</td>
<td>5.8</td>
<td>11.0</td>
<td>1.2</td>
<td>6.6</td>
<td>5.8</td>
</tr>
</tbody>
</table>
Quarkonia survival probability in HIC

\[ S = \frac{\int_0^\infty d\tau P(\tau) \exp \left( - \int_\tau^{\tau_c} \Gamma(\tau')d\tau' \right)}{\int_0^\infty d\tau P(\tau)} \]

Formation time enhances the survival probability of quarkonia in HIC
Summary

- Quarkonium formation time affects its production in HIC since dissociation effects occur after formation.
- Most current studies of quarkonium nuclear modification factor treat formation time as a parameter or use its values in vacuum.
- Quarkonium formation time increases in QGP due to changes in mass and size.
- Larger quarkonium formation time in QGP enhances its survival probability in HIC.
- Further studies are needed for the formation time of color octet initial state and in thermal regeneration.