Jet Structure in Heavy Ion Collisions

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Jets in QCD

A jet is an energetic and collimated bunch of particles produced in a high-energy collision.

LEP (OPAL) events

1993

\[ Q^2 = s \]
Jets originate from energetic partons that successively branch (similarly to an accelerated electron that radiates photons).

Elementary branching process is enhanced in the collinear region $\implies$ Collimated jets.

\[ dP \sim \alpha_s C_R \frac{d\theta}{\theta} \frac{d\omega}{\omega} \]
Jets substructure in vacuum

[Bassetto, Ciafaloni, Marchesini, Mueller, Dokshitzer, Khoze, Troyan,…80']

The jet is a **coherent** object, successive branchings are ordered from larger to smaller angles $\theta_1 > \theta_2 > \ldots > \theta_n$

(coherence leads to destructive interferences between radiations at large angles)

\[ Q \equiv E \theta_{\text{jet}} \]

Jet transverse mass

\[ Q_0 \sim \Lambda_{\text{QCD}} \]

hadrons

partons
Jets in Heavy Ion collisions
Jets in HIC are more involved than in vacuum

- Experimental jet definition depends on experimental procedure: jet reconstruction algorithm, background subtraction, unfolding, etc. But to what extend?


- Q: is relevant physics under control? Monte Carlo Event-Generators are available or under construction: MARTINI, JEWEL, Q-PYTHIA, PYQUEN/HYDJET, YAJEM, MATTER
Medium-induced QCD cascade and angular broadening

\[ \omega \text{ and } \theta \]
Medium-induced radiation


- Radiation triggered by multiple scatterings, related to momentum broadening: \( \langle k_{\perp}^2 \rangle = \hat{q} \, t_f \)

\[
\frac{\omega}{k_{\perp}^2} \sim \sqrt{\frac{\omega}{\hat{q}}} \]

Coherent radiation

formation time \( t_f = \frac{\omega}{k_{\perp}^2} \)
Medium-induced radiation

- Landau-Pomeranchuk-Migdal suppression

\[ \omega \frac{dN}{d\omega} = \alpha_s \frac{L}{t_f} \equiv \alpha_s N_{\text{eff}} \]
\[ \omega \frac{dN}{d\omega} \equiv \frac{L}{t_*(\omega)} \]

- Maximum suppression when \( t_f > L \) or \( \omega > \omega_c = \hat{q}L^2 \)

- Minimum angle \( \theta > \theta_c \equiv 1/\sqrt{\hat{q}L^3} \)

- Characteristic time scale: effective inelastic mean free path

\[ t_*(\omega) \sim \frac{1}{\alpha_s t_f(\omega)} \]

- Multiple (independent) branching regime: \( t_f \ll t_* \ll L \)
Evolution of the gluon distribution

- **Coll. branching approx:** Momentum broadening during the branching process is suppressed \( t_f \ll t_* \Rightarrow k_f^2 \ll k_*^2 \)

\[
\frac{\partial}{\partial t} D(\omega, \theta) = \int_0^1 dz K(z) \left[ \frac{D(\omega/z, \theta)}{t_*(\omega/z)} - \frac{D(\omega, \theta)}{t_*(\omega)} \right] - \frac{\hat{q}}{\omega^2} \nabla^2_{\theta} D(\omega, \theta)
\]

\( D(\omega, \theta) \equiv \omega \frac{dN}{d\omega d\theta^2} \)

**Gain** \( \omega/z \)

**Loss** \( \omega/z \)

Diffusion btw 2 branchings

**Energy loss:** Baier, Mueller, Schiff, Son [2001], Moore, Jeon [2003]

**Energy recovery:** Blaizot, Dominguez, Iancu, MT [2013]

**Splitting kernel**

\[ K(z) \sim \frac{1}{z^{3/2}(1-z)^{3/2}} \]

**Inelastic mean-free-path**

\[
t_*(\omega) = \frac{1}{\alpha_s} \sqrt{\frac{\omega}{\hat{q}}}
\]
Evolution of the gluon distribution

- **Coll. branching approx**: Momentum broadening during the branching process is suppressed \( t_f \ll t_\ast \Rightarrow k_f^2 \ll k_\ast^2 \)

\[
\frac{\partial}{\partial t} D(\omega, \theta) = \int_0^1 dz K(z, \hat{q}) \left[ \frac{D(\omega/z, \theta)}{t_\ast(\omega/z)} - \frac{D(\omega, \theta)}{t_\ast(\omega)} \right] - \frac{\hat{q}}{\omega^2} \nabla^2_\theta D(\omega, \theta)
\]

Broadening due to branchings logarithmically enhanced


This large correction can be fully absorbed in a renormalization of the quenching parameter

\[
\hat{q} \equiv \hat{q}_0 \left( 1 + \frac{2 \alpha_s N_c}{\pi} \ln^2 \frac{k_\perp^2}{m_D^2} \right)
\]

Blaizot, MT (2014)
Universality of the double logs

As a consequence to the renormalization of the quenching parameter, the DL's not only enhance the \textit{pt-broadening} but also the \textit{radiative energy loss} expectation:

For large media (asymptotic behavior)

\[
\langle k_{\perp}^2 \rangle \propto L^{1+\gamma} \quad \text{anomalous dimension}
\]

\[
\Delta E \propto L^{2+\gamma}
\]

\[
\gamma = \sqrt{\frac{4\alpha_s N_c}{\pi}}
\]

To be compared to N=4 SYM (strong-coupling) estimate

\[
\Delta E \sim L^3
\]

[Gubser et al, Hatta et al, Chesler, Yaffe (2008)]
Energy flow and wave turbulence

- Scaling solution in the soft sector $\omega < \omega_s \ll E$
- Constant flow of energy down to the medium temperature scale $\omega \sim T$
- Parametrically large angle $\theta_*(\omega) \equiv \frac{k_{*\perp}}{\omega} > \frac{1}{\alpha_s^2} \theta_c \gg \theta_{jet}$ with $k_{*\perp}^2 \equiv \hat{q} t_*$

\[ \Delta E \sim \omega_s \sim \alpha_s^2 \omega_c \]

Gluon distribution as function of time

\[ D(\omega) \sim 1/\sqrt{\omega} \]
Angular distribution

I — Leading particle \( \omega \sim E \)

II — Rare BDMPS radiation: \( \omega_s \ll \omega \ll \omega_c < E \)

III — Multiple branching regime \( \omega \ll \omega_s \ll E \)
I — Leading particle $\omega \sim E$

- Not sensitive to gluon radiation (or only via the renormalized quenching parameter)

$$D(\omega, \theta) \sim D(\omega) \mathcal{P}(\theta, \omega) \sim \omega \delta(\omega - E) \mathcal{P}(\theta, \omega)$$

- Broadening Probability

$$\mathcal{P}(\theta, \omega) \equiv \frac{4\pi}{\langle \theta \rangle^2} e^{-\theta^2 / \langle \theta \rangle^2}$$

$$\langle \theta \rangle^2 \equiv \frac{\hat{q} L}{E^2} \quad (\sim 0.001)$$

The deflection of the jet is negligible!
II — Rare BDMPS radiation: $\omega_s \ll \omega \ll \omega_c < E$

Single gluon radiation $O(\alpha_s)$. The broadening is determined by the diffusion of the radiated gluon

$$D(\omega, \theta) \simeq \alpha_s \int_0^L dt \, \mathcal{P}(\theta, \omega, L - t) \sqrt{\frac{\hat{q}}{\omega}}$$

The typical angular broadening reads (the factor $1/2$ comes from the time integral)

$$\langle \theta^2 \rangle \equiv \frac{\langle k_{\perp}^2 \rangle}{\omega^2} = \frac{\hat{q}L}{2\omega^2} > \theta_c^2$$
III — Multiple branching regime $\omega \ll \omega_s \ll E$

Multiple branching + multiple scatterings

Soft gluon cascades take place at parametrically large angle

$$\theta \gg \frac{1}{\alpha_s^2 \theta_c} \gg \theta_c$$
III — Multiple branching regime \( \omega \ll \omega_s \ll E \)

The solution can be written in the factorized form

\[
D(\omega, \theta) = D(\omega) \eta(\theta^2 / \theta_*^2(\omega))
\]

where \( \theta_*^2(\omega) = \frac{1}{\alpha_s} \left( \frac{\hat{q}}{\omega} \right)^{1/2} \)

The normalized angular distribution (at all orders in opacity expansion)

\[
\eta(z) = \int_0^\infty d\beta J_0(2\sqrt{z\beta}) \sum_{n=0}^{\infty} (-1)^n c_n \beta^{2n} \quad z = \theta^2 / \theta_*^2
\]
Partial decoherence of intrajet structure
Partial decoherence of intrajet structure

- Consider two subsequent splittings

- Color coherence yields the angular ordering constraint on the second branching

\[ dP_2 \sim \alpha_s \frac{d\theta}{\theta} \frac{d\omega}{\omega} \Theta(\theta_1 - \theta) \]
Partial decoherence of intrajet structure

[MT, Salgado, Tywoniuk (2010-2011) Iancu, Casalderray-Solana (2011) ]

• Consider two subsequent splittings

\[ \Delta_{\text{med}} \equiv 1 - e^{-\hat{q} L r_{\perp}^2 / 12} \]

\[ r_{\perp} \sim \theta_1 L \]

• The interaction with the medium alters angular ordering

\[ dP_2 \sim \alpha_s \frac{d\theta}{\theta} \frac{d\omega}{\omega} \left[ \Theta(\theta_1 - \theta) + \Delta_{\text{med}} \Theta(\theta - \theta_1) \right] \]

coherence: \( \Delta_{\text{med}} \rightarrow 0 \)  

decoherence: \( \Delta_{\text{med}} \rightarrow 1 \)
A multi-scale problem

How to treat Interferences between Vacuum and in-medium shower?

Jet mass
\[ Q \equiv \theta_{\text{jet}} \, E \]
\[ Q_0 \sim \Lambda_{\text{QCD}} \]
pt-broadadeding
\[ Q_{\text{med}} \equiv (\hat{q}L)^{1/2} \]
Jet size
\[ r_{\perp}^{-1} \equiv (\theta_{\text{jet}} L)^{-1} \]
Transparency limit: Jets as coherent objects

\[ Q \gg r_{\perp}^{-1} \gg Q_{\text{med}} \gg Q_0 \]

The medium interacts mostly with the total charge (original parton)

\[ x = \frac{E_{\text{parton}}}{E} \]

\[ Q \equiv p_{\perp} \Theta_{\text{jet}} \]

In-cone parton dist: Convolution of in-medium and vacuum evolution

\[ x \frac{dN}{dx} \equiv D^{\text{coh}}(x) \equiv \int_{x}^{1} \frac{d\bar{z}}{\bar{z}} D_{\text{vac}}(x/\bar{z}, Q) D_{\text{med}}(\bar{z}, p_{\perp}) \]

Corrections due to partial decoherence of vacuum radiation \sim r^2

\[ D^{\text{tot}} \equiv D^{\text{coh}} + \Delta D^{\text{decoh}} \]
Theory vs. data

(I) Missing $p_T$ in asymmetric dijet events

(II) Medium-modified Fragmentation Function

(1) Missing $p_T$ in dijet events

- Selection of dijet events with large energy Imbalance $p_{T1}>120$ GeV and $p_{T2}>50$ GeV

CMS: energy is lost in soft particles at large angles
(1) Missing $p_T$ in dijet events

Momentum imbalance

![Graph showing momentum imbalance with and without IR-cutoff]

Cumulative Energy

CMS DATA

![Graph showing CMS data for PbPb events at $\sqrt{s_{NN}}=2.76$ TeV]

Leading jet: $L_1 = 1$ fm, Subleading jet: $L_2 = 5$ fm,

with $E = 120$ GeV, $\hat{q} = 2$ GeV$^2$/fm, $\alpha_s = 0.3$
(II) Fragmentation Functions

- vacuum baseline (blue)
- medium-induced energy loss at large angles depletes energy inside the cone, responsible for dip in the ratio
- small angle soft radiation due to decoherence of vacuum radiation: possibly responsible for enhancement at large $l = \text{shift of humpbacked plateau!}$

Nota: the different suppression of quark and gluon jets does not explain the soft enhancement [M. Spousta, B. Cole (2015)]
Summary and outlook

- Jets in HIC are composed of a coherent inner core and large angle decoherent gluon cascades that are characterized by a constant energy flow from large to low momenta down to the QCD scale where energy is dissipated. Geometrical separation: The cascade develop at parametrically large angles away from the jet axis.

- Excess of soft particles within the jet cone might be a signature of color decoherence.

- Comparison with data: consistent picture, but need a Complete Monte Carlo Event Generator (to deal with experimental biases) and a realistic treatment of the geometry of the collision, particle content, hadronization, etc, for quantitative studies.
Jets in QCD

• Jet ~ parton produced in a hard process at high energy: large separation between short and large distance physics: $p_T \gg \Lambda_{\text{QCD}}$ (QCD-factorization)

• Experimentally jets are reconstructed by clustering particles, whose total energy is $p_T$, within a given cone of size $R$. An approximate way of reversing the process of fragmentation and compare to theory
Jets substructure in vacuum

Fragmentation Function

$$D(x) = x \frac{dN}{dx}$$

- Perturbative QCD prediction for the distribution of hadrons in a jet
- 2 scales: non-perturbative scale $Q_0 = \Lambda_{QCD}$ and the jet transverse scale $= E \theta_{jet}$
- Angular Ordering (AO) $\Rightarrow$ soft gluon emissions (large $l \sim$ small $x$) are suppressed

$$l \equiv \ln \frac{1}{x} \quad x \equiv \frac{E_h}{E_{jet}}$$

[Okhtserin, Khoze, Mueller, Troyan, Kuraev, Fong, Webber...80']
First $\alpha_s$ correction enhanced by a double log (DL) is resummed with strong ordering in formation time (or energy) and transverse mom. of overlapping successive gluon emissions.

Renormalization Group Equation for the quenching parameter

$$\frac{\partial \hat{q}(\tau, Q^2)}{\partial \ln(\tau/\tau_0)} = \int_{\hat{q}_\tau}^{Q^2} \bar{\alpha}(\hat{q}) \frac{dq^2}{q^2} \hat{q}(\tau, q^2)$$

with initial condition

$$\hat{q}(\tau_0) \equiv \hat{q}_0$$
The evolution equation may be solved in Fourier space 
($r_T \sim u_T/\omega \sim$ transverse dipole size)

\[ D(\omega, u) \equiv \int d^2 \theta \, D(\omega, \theta) \, e^{-iu \cdot \theta} \]

Opacity Expansion: (order by order in elastic scatterings but all order in branchings)

\[ D(\omega, u) = \sum_{n=0}^{\infty} D_n(\omega, u), \]
III — Multiple branching regime $\omega \ll \omega_s \ll E$

The general term reads

$$D_n(\omega, u) = c_n \left[ \sigma(\omega, u) t_*(\omega) \right]^n D(\omega)$$

where the coefficients $c_n$ are solved recursively

$$c_n = \prod_{m=1}^{n} \frac{2}{\sqrt{\pi}} \frac{\Gamma\left(\frac{3m}{2}\right)}{\Gamma\left(\frac{3m+1}{2}\right)}$$

Why $t_*(\omega)$ and not $L$? A gluon $\omega$ can not survive in the medium longer than $t_*(\omega)$ therefore, to be measured it must be produced close to the surface within the shell $L - t_*(\omega)$
(1) Nuclear Modification Factor

\[ R_{AA} = \frac{dN_{AA}}{dN_{pp} \times N_{coll}} \]

\[ \frac{dN_{jet}^{AA}}{d^2 p_\perp} \equiv N_{coll} \int_0^1 \frac{dx}{x} D_{med}(x, p_\perp/x) \frac{dN_{jet}^{pp}}{d^2 p_\perp} (p_\perp/x) \]

Solving the evolution equation for D convolved with an initial power spectrum \( p_{-n} \)

\[ L = 2 - 3 \text{ fm} \]
\[ \hat{q} = 2.5 - 6 \text{ GeV}^2/\text{fm} \]