QCD Equation of State and Hadron Resonance Gas

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arXiv:0912:2541
Hadron Resonance Gas model

- EoS of interacting hadron gas well approximated by non-interacting gas of hadrons and resonances

\[
P(T) = \sum_i \int d^3p \frac{p^2}{3E} f(p, T)
\]

- valid when
  - interactions mediated by resonances

- Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts
  \( \Rightarrow \) HRG good approximation at low temperatures
  \( \rightarrow \) lattice should reproduce HRG at \( T \leq 120 - 140 \) MeV

- practical problem: how to convert fluid to particles?
- energy conservation iff EoS is the same before and after freeze-out
EoS by hotQCD collaboration

Bazavov et al. arXiv:0903.4379 [hep-lat]

- evaluate interaction measure \( (\epsilon - 3P)/T^4 \)
- obtain pressure via

\[
\frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\epsilon - 3P}{T'^5}
\]

- What is \( P(T_0) \)?
- What is \( (\epsilon(T_0) - 3P(T_0))/T_0^4 \)?
- How good is lattice below \( T_c \)?
Trace anomaly below $T_c$

Bazavov et al arXiv:0903.4379

$\frac{(\varepsilon-3p)}{T^4}$

- asqtad: $N_\tau=8$
- $p4$: $N_\tau=8$

- Lattice EoS $\neq$ Hadron Resonance Gas EoS
Hadrons on lattice

- Hadron masses depend on lattice cutoff
  \[ \Rightarrow \text{i.e. on temperature:} \]
  
  E.g. for pseudoscalar mesons

\[
m_{ps_i}^2 = m_{ps0}^2 + \frac{1}{r_1^2} \left( a_{ps}^i x + b_{ps}^i x^2 \right) \frac{1}{(1 + c_{ps}^i x)^\beta_i}
\]

\[
x = \left( a/r_1 \right)^2
\]

\[
a = \frac{1}{N \tau T}
\]

+ 16 pseudoscalar mesons on lattice

- HRG with lattice mass spectrum?
Hadronic fluctuations

i.e. baryon number, strangeness and charge susceptibilities

\[ \chi_2^x = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial (\mu_x/T)^2} = \frac{1}{T^2} \frac{\partial^2 P}{\partial \mu_x^2}, \]

where \( \mu_x = \mu_B, \mu_S \) or \( \mu_Q \)

- Lattice masses \( \rightarrow \) fluctuations in resonance gas and lattice similar
• very little room for modifications in hadron gas
• **BUT**, what is physical mass spectrum?
• **conservative estimate**: free particle masses
Phenomenological EoS

- $T < T_{sw}$: HRG interaction measure (black)
- $T > T_{sw}$: Lattice interaction measure (red)

- $\epsilon$ and $P$ overshoot Stefan-Boltzmann limit!
- Interaction measure too large, but where?
Interaction measure

Cheng et al ('08)

Bazavov et al ('09)

- peak region sensitive to $N_\tau$
Procedure for EoS

• HRG below \( T \approx 180 - 190 \text{ MeV} \)

• Parametrize lattice using:

\[
\frac{\epsilon - 3P}{T^4} = \frac{d_2}{T^2} + \frac{d_4}{T^4} + \frac{c_1}{T^{n_1}} + \frac{c_2}{T^{n_2}}
\]

• Require that:

\[
\frac{\epsilon - 3P}{T^4} \bigg|_{T_0}, \quad \frac{d}{dT} \frac{\epsilon - 3P}{T^4} \bigg|_{T_0}, \quad \frac{d^2}{dT^2} \frac{\epsilon - 3P}{T^4} \bigg|_{T_0}
\]

are continuous \( (1) \)

\[
\frac{P}{T^4} \bigg|_{T=800\text{MeV}} = 0.95 \frac{s_{SB}}{T^4}
\]

\( \Rightarrow T_0, \ d_4, \ c_1, \ c_2 \text{ fixed} \)

• \( \chi^2 \text{ fit to lattice above } T = 250 \text{ MeV} + \text{ one point at } T = 206 \text{ MeV} \)

• We get \( T_0 = 183.8 \text{ MeV}, \ d_2 = 0.2660, \ d_4 = 2.403 \cdot 10^{-3}, \)
  \( c_1 = -2.809 \cdot 10^{-7}, \ c_2 = 6.073 \cdot 10^{-23}, \ n_1 = 10, \ n_2 = 30 \)
• obtain pressure via

\[
\frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\epsilon - 3P}{T'^5}
\]
Speed of sound

- no softening below the HRG!

\[ \text{no softening below the HRG!} \]
Bazavov and Petreczky, arXiv:1005.1131

\[ \frac{(\varepsilon - 3p)}{T^4} \]

- HISQ, \( N_f = 8 \)
- HISQ, \( N_f = 6 \)
- asqtad
- p4
- Laine
- s95p-v1

\[ T \ [\text{MeV}] \]

\[ \text{Data Points} \]

\[ \text{Curves} \]
Bazavov and Petreczky, arXiv:1005.1131

\(\frac{(\varepsilon-3p)}{T^4}\)

- HISQ, \(N_t=8\)
- HISQ, \(N_t=6\)
- asqtad
- \(p4\)
- Laine
- s95p-v1

HISQ, \(N_t=6\)

\(\frac{(\varepsilon-3p)}{T^4}\)

- HRG physical
- HRG distorted stout \(N_t=8\)
- HRG distorted asqtad \(N_t=8\)

- stout \(N_t=10\)
- stout \(N_t=8\)
- asqtad \(N_t=8\)
- \(p4\) \(N_t=8\)
Ideal hydrodynamics

matter in local equilibrium: \( T^{\mu\nu} = (e + p)u^\mu u^\nu - pg^{\mu\nu} \), \( N^\mu = nu^\mu \)

local, macroscopic variables:
- energy density \( e(x) \)
- pressure \( p(x) \)
- flow velocity \( u^\mu(x) \) \((u^\mu u_\mu = 1)\)
- baryon density \( n(x) \)

energy-momentum and charge conservation:

\[
\partial_\mu T^{\mu\nu}(x) = 0 \\
\partial_\mu N^\mu(x) = 0
\]

Unknowns: initial state, final state

matter characterized by: equation of state \( p(e, n) \)
Elliptic flow $v_2$

Spatial anisotropy $\rightarrow$ Final azimuthal momentum anisotropy

$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

Sensitive to speed of sound $c_s^2 = \partial p / \partial e$ and shear viscosity $\eta$
Effect on flow I

- ideal fluid, $b = 7$ fm
- keep everything fixed:
  - $\tau_0 = 0.6$ fm/$c$, $T_{dec} = 125$ MeV

$\implies$ harder EoS, flatter spectra
Effect on flow II

• ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV
• chemical equilibrium

- $T_{dec} = 140$ MeV
- EoS Q: first order phase transition at $T_c = 170$ MeV, $T_{dec} = 125$ MeV
Chemical non-equilibrium

- ideal fluid, \( b = 7 \ \text{fm} \)
- keep everything fixed:
- \( \tau_0 = 0.2 \ \text{fm}/c, \ T_{\text{chem}} = 150 \ \text{MeV}, \ T_{\text{dec}} = 120 \ \text{MeV} \)

\[ \Rightarrow \text{harder EoS, flatter spectra} \]
• ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV  
• $T_{\text{chem}} = 150$ MeV

\begin{itemize}
  \item EoS Q: $T_{dec} = 120$ MeV, $s_{\text{ini}} \propto N_{\text{bin}}$, $\tau_0 = 0.2$ fm/c
  \item s95p, $\tau_0 = 0.8$: $T_{dec} = 120$ MeV, $s_{\text{ini}} \propto N_{\text{bin}}$, $\tau_0 = 0.8$ fm/c
  \item s95p, $\tau_0 = 0.2$: $T_{dec} = 120$ MeV, $s_{\text{ini}} \propto N_{\text{bin}} + N_{\text{part}}$, $\tau_0 = 0.2$ fm/c
\end{itemize}
Conclusions

- below $T_c$ lattice and HRG differ because of hadron mass spectrum

$\Rightarrow$ HRG good description below $T_c$

- some uncertainty in the parametrization of the EoS

$\Rightarrow$ but it doesn’t matter

- proton $v_2(p_T)$ may or may not be sensitive to EoS — details matter!

- EoS tables available at

  and