Cosmological QCD Phase Transition and Little Inflation

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and Simon Schettler (to be published)
Prelude: the bubbling universe
Breaking news: violence in the universe!

New Scientist, issue of May 22, 2010
REPAIRING REALITY
Computer games to save the world

TASTE OF TINY
The nanofood revolution

INFLATION II
Contains scenes of significant violence

NewScientist

SECRETS OF THE ICE AGES
Earth’s roller-coaster climate explained

FREE!
10 BEST JOBS IN SCIENCE
Big bang, part II: the big boil

Big bang II: a second inflation, and a settling mass of stuff as follows the universe's trail

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The big boil

A big uncertainty is how long the universe would expand and cool before bubbles began to form. The answer could be just a few thousand years, or even less than a billion. The outcome is uncertain because of the way the expansion is governed by two equations: the Hubble expansion and the radiation domination. The Hubble expansion is governed by the relation between the Hubble parameter, H, and the Friedmann parameter, f, which represents the ratio of the energy density of the universe to the critical density. The radiation domination is governed by the relation between the temperature of the universe, T, and the energy density of the universe, ρ, which represents the ratio of the energy density of the universe to the critical density. These equations are coupled by the equations of state of the universe. At high temperatures, the coupled equations are satisfied by a single equation: the radiation-neutrino equation, which describes the energy density of the universe and the temperature of the universe in terms of the number density of the universe and the temperature of the universe. This equation is satisfied by a single equation: the radiation-neutrino equation, which describes the energy density of the universe and the temperature of the universe in terms of the number density of the universe and the temperature of the universe. At low temperatures, the coupled equations are satisfied by a single equation: the radiation-neutrino equation, which describes the energy density of the universe and the temperature of the universe in terms of the number density of the universe and the temperature of the universe.
Big bang II: the big boil, the final words . . .

This Week

Big bang, part II: the big boil

The universe may well have bubbled, but for now the idea is speculative, says Schaefer . . .
Just a phase the universe went through

As the early universe cooled, the quark-gluon plasma underwent either a smooth or a "bubbling" phase change to form the matter we see today. Experiments are set to probe the transition at various ratios of matter to antimatter.

LHC: Large Hadron Collider
RHIC: Relativistic Heavy Ion Collider
FAIR: Facility for Antiproton and Ion Research
Outline

- Standard cosmology
- QCD phase transition with a little inflation
  Ingredients: Affleck-Dine baryogenesis, metastable vacuum, bubble nucleation
- Implications and possible signals:
  - large-scale structure
  - WIMPs and mini black holes
  - cosmological magnetic fields
  - gravitational wave background
- Summary
Early universe: temperature increases with scale parameter as $a^{-1}$

- at $t = 1\text{s}$ to 3 minutes: BBN ($T = 0.1$ to $1\text{ MeV}$)
- at $t \approx 10^{-5}\text{s}$: QCD phase transition ($T \approx 170\text{ MeV}$)
- at $t \approx 10^{-10}\text{s}$: electroweak phase transition ($T \approx 100\text{ GeV}$)
Phase Transitions in QCD

- early universe at small baryon density and high temperature
- neutron star matter at small temperature and high density
- first order phase transition at high density
- probed by heavy-ion collisions with CBM@FAIR!
Standard cosmology

from microwave background radiation and big bang nucleosynthesis:

\[ \frac{n_B}{s} \sim \frac{n_B}{n_\gamma} \sim \frac{\mu}{T} \sim 10^{-9} \]

note: baryon number per entropy is conserved

\[ \Rightarrow \text{early universe evolves along } \frac{\mu}{T} \sim 10^{-9} \sim 0 \]

\[ \Rightarrow \text{crossover transition, nothing spectacular, no cosmological signals} \]

Friedmann equation for radiation dominated universe:

\[ H^2 = \frac{8\pi G}{3} \rho \sim g(T) \frac{T^4}{M_p^2} \]

\( g(T) \): effective number of relativistic degrees of freedom at \( T \)

Hubble time (true time \( t = 3t_H \) for radiation dominated universe):

\[ t_H = \frac{1}{H} \sim g^{-1/2} \frac{M_P}{T^2} \Rightarrow \frac{t}{1 \text{ sec}} \sim \left( \frac{1 \text{ MeV}}{T} \right)^2 \]
A little inflation at the QCD phase transition

what happens if the early universe passes through a first order phase transition?

- is this possible? \( \Rightarrow \) Yes! no contradiction with present data
- could this be observable? \( \Rightarrow \) Yes! by gravitational waves

1st order phase transition \( \Rightarrow \) false metastable vacuum
\( \Rightarrow \) de Sitter solution \( \Rightarrow \) (additional small) inflationary period

\[ H = \dot{a}/a \sim M_p^{-1} \rho_v^{1/2} = H_v = \text{const.} \quad \rightarrow \quad a \sim \exp(H_v \cdot t) \]

just a few e-folds are enough (standard inflation needs \( N \sim 50 \)):

\[ \left( \frac{\mu}{T} \right)_f \approx \left( \frac{a_i}{a_f} \right)^3 \left( \frac{\mu}{T} \right)_i \]

Hence \( \mu/T)_i \sim \mathcal{O}(1) \) for just \( N = \ln \left( a_f/a_i \right) \sim \ln(10^3) \sim 7 \) e-folds

(first order phase transition by a large lepton asymmetry: Schwarz, Stuke 2009)
First-order phase transition: linear $\sigma$ model

potential within the linear $\sigma$ model at finite temperature

left plot: high $T$, right plot: low $T$, system being trapped in false vacuum state

possibility of a ’quench’ at finite $\mu$, two scalar fields in QCD – hybrid inflation?
A little inflation in the QCD phase diagram

(Boeckel and JSB, arXiv:0906.4520)

- start with $\mu/T \sim 1$ (possible for e.g. Affleck-Dine baryogenesis)
- universe trapped in false vacuum at the transition line
- supercooling and dilution with $\mu/T = \text{const}$.
- decay to the true vacuum state $\rightarrow$ reheating to $T \sim T_c$ so that $\mu/T \sim 10^{-9}$
- then standard cosmological evolution to BBN
A little inflation – evolution of densities

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- Energy density falls as \( a^{-4} \) until \( \rho \sim \Lambda_{\text{QCD}}^4 \)
- Then \( \rho = \text{const.} \) → inflationary period starts
- Reheating at the end of inflation
- Maximum length of inflation for scale parameter \( a \) from CDM density \( \sim 10^3 \)

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(Boeckel and JSB, arXiv:0906.4520)
End of phase transition by bubble nucleation

bubble of new phase grows if they exceed a critical bubble size
free energy:
\[ \Delta F = -\frac{4\pi}{3} R^3 \Delta p + 4\pi R^2 \sigma \]

with a critical bubble size of \( R_c = \frac{2\sigma}{\Delta p} \), nucleation rate:
\[ \Gamma = P_0 \exp \left( -\frac{\Delta F}{T} \right) \text{ with } P_0 \sim T^4 \]
depends crucially on surface tension \( \sigma \) and pressure difference \( \Delta p \):
\[
\frac{\Delta F_c}{T} = \frac{16\pi \sigma^3}{3T(\Delta p)^2} = \frac{16\pi}{3} \left( \frac{\sigma}{200 \text{ MeVfm}^{-2}} \right)^3 \left( \frac{200 \text{ MeV}}{T} \right) \left( \frac{200 \text{ MeVfm}^{-3}}{\Delta p} \right)^2
\]

exponential suppression!
in general \( \sigma = \sigma(T) \) as the barrier vanishes for low \( T \)
(ensures a graceful exit!)
failure to nucleate $\tau_{\text{nucl}} > t_{\text{hubble}}$ for $\sigma > 120$ MeV/fm$^2$

(MIT bag model with $B^{1/4} = 145$ MeV)

(Jenkovszky, Sysoev, Kämpfer 1990; Csernai, Kapusta 1992; Mintz, Fraga, Pagliara, JSB 2010)

surface tension in QCD: $\sigma = 50 - 150$ MeV/fm$^2$ or smaller or larger . . .

(Voskresensky, Yasuhira, Tatsumi 2003; Palhares, Fraga 2010)
Power Spectrum of Dark Matter

- dark matter mass within horizon at $T_c \approx 170$ MeV: $10^{-9} M_\odot$

- boosted by little inflation by $(a_f/a_i)^3 \approx 10^9$ so that mass scales of up to $1 M_\odot$ are affected

- additional effect for modes $k_{ph} < H$ at the beginning of inflation

- two scales involved: $H^2 \propto \rho_v \sim \text{const.}$ and

\[
\ddot{H} = -4\pi G (\rho + p) = -4\pi G (\rho_{dm} + 4\rho_r/3) \propto \left(a_i/a\right)^q \quad \text{where} \quad q = 3 \ldots 4
\]

- three spectral regimes:
  - $(k_{ph}/H)_i > a_f/a_i$: always subhubble
  - $a_f/a_i > (k_{ph}/H)_i > (a_i/a_f)^{q/2}$: intermediate
  - $(k_{ph}/H)_i < (a_i/a_f)^{q/2}$: unaffected

- highest mass scale affected is

\[
M_{\text{max}} \sim 10^{-8} M_\odot (a_f/a_i)^{3q/2} \sim (10^6 - 10^8) M_\odot
\]

- relation to cuspy core, subhalo issues of structure formation?
freeze-out of weakly interacting massive particles (WIMPs):

$$\Omega_{CDM} \sim \sigma_{\text{weak}} / \sigma_{\text{ann}}.$$ 

$$\rho_{CDM}$$ will be larger by $$(a_f/a_i)^3$$ during freeze-out before inflation

need substantially reduced annihilation cross section, correspondingly reduced production cross section

can be checked @LHC! (if SUSY particles are not found)

primordial black hole production due to collapsing bubbles:

$$M_{bh} \sim M_{\text{hubble}} \sim 1M_\odot$$

as the total energy density after inflation is involved

(Jedamzik 1997; Kapusta, Springer 2007)
Seeds for magnetic fields

- primordial magnetic fields produced by bubble collisions in first order phase transition (Cheng, Olinto 1994)
- charge dipole layer at surface, high baryon density contrast
- magnetic field can be $B_{QCD} \sim 10^8 - 10^{10}$ G
- amplified by MHD turbulence to equipartition value
  $$B_{eq} = \sqrt{8\pi T^4 v_f^2} \sim 10^{12}$$ G (Sigl, Olinto, Jedamzik 1997)
- little inflation scenario boosts magnetic fields by higher density and larger baryon diffusion length
- can explain presently observed (extra)galactic magnetic field strength $B_{obs} \sim 0.1 - 1 \mu$G
- works for GUT and QCD phase transition
  (Caprini, Durrer, Fenn 2009)
Tensor perturbations and QCD trace anomaly

- crucial input for tensor perturbations in GR: trace anomaly of QCD!

- EoM for tensor perturbation amplitude $v_k = a \cdot h_k$ in Fourier space (gauge invariant):

$$v_k''(\eta) + \left( k^2 - \frac{a''}{a} \right) v_k(\eta) = 0$$

where

$$\frac{a''}{a} = \frac{4\pi G a^2}{3} (\rho - 3p)$$

- only input needed: QCD trace anomaly

- use several lattice parameterizations, compare with simple bag model
parameterization of lattice data with improved staggered fermion actions (asqtad and p4) and physical strange quark masses, with and without a hadron resonance gas (HRG)

(Bazavov et al., Bielefeld-BNL/RIKEN-Columbia collaboration 2009)
Gravitational wave background from QCD phase transition

- energy density in gravitational wave background: \( \Omega_g(k) = \frac{1}{\rho_c} \frac{d \rho_g}{d \ln k} \)
- mode \( h_k \) is damped by \( 1/a \) after horizon entry
- entropy conservation: \( g a^3 T^3 = \text{const.} \rightarrow H \sim T^2 g^{1/2} \sim g^{-1/6} a^{-2} \)
- as \( \Omega_g \sim H_{in}^2 a_{in}^4 \sim g_k^{-1/3} \), so

\[
\frac{\Omega_g(\nu \gg \nu^*)}{\Omega_g(\nu \ll \nu^*)} = \left( \frac{g_f}{g_i} \right)^{1/3} \sim 0.7
\]

(Schwarz 1998)

- step in amplitude at frequency scale given by (redshifted) horizon scale at the transition point

\[
\nu_{\text{peak}} \sim H_c \cdot T_{\gamma,0}/T_c \sim T_c/M_p \cdot T_{\gamma,0} \sim 10^{-7} \text{ Hz}
\]

- maximum amplitude \( h \sim a/a_0 \sim 10^{-12} \)
A step in the gravitational wave background

- step in gravitational wave background around $\nu \sim 10^{-7}$ Hz
- step in spectrum of about $(g_f/g_i)^{1/3} \sim 0.7$
- rather insensitive to details of the phase transition (Schwarz 1998)
amplitudes are exponentially suppressed during inflation as $h \sim 1/a \sim \exp(H \cdot t)$

gravitational wave background drops as $\nu^{-4}$
Observations of gravitational wave background

- gravitational wave amplitude versus frequency
- gravitational waves measurable with pulsar timing (PPTA and SKA) or space-based interferometers (LISA)
- step frequency in the amplitude close to highest sensitivity for pulsar timing
Gravitational waves from bubble collisions

- First order transition produces tensor perturbations $\rightarrow$ gravitational waves

- Amplitude scales as $h(\nu) \propto \nu^{-1/2}$ for $\nu < H$ (white noise)

- and as $h(\nu) \propto \nu^{-2\ldots-1}$ for $\nu > H$ (multi bubble collisions)

(Kamionkowski, Kosowsky, Turner 1994; Huber, Konstandin 2008)
Producing gravitational waves with bubbles

energy emitted in gravitational waves (quadrupole formula):

\[ E_{GW} \sim G \frac{Q^2}{\tau} \]

with duration of collision \( \tau \) and separation of bubbles \( d \sim \tau \)

\[ Q \sim \frac{\rho_v \cdot d^3 \cdot \tau^2}{\tau^3} \sim \rho_v \tau^2 \]

energy relative to total energy:

\[ \frac{E_{GW}}{E_v} \sim \frac{G \rho_v^2 \tau^2}{\rho_v \tau^3} \sim G \rho_v \tau^2 \sim \left( \frac{\tau}{H^{-1}} \right)^2 \]

limit from Parkes Pulsar Timing Array PPTA: \( \tau / H^{-1} < 0.12 \)

will be improved by full PPTA data set and by Square Kilometre Array SKA in the future
Summary

- first order transition could have happened in the early universe
- need large initial $\mu/T$ and a metastable false vacuum state
- large-scale structure modified up to $M \sim 10^9 M_\odot$
  (without QCD inflation only up the horizon mass $\sim 10^{-9} M_\odot$)
- cold dark matter density is diluted by $10^{-9}$
  $\rightarrow$ need different WIMP annihilation cross section as $\Omega_{CDM} \sim \sigma_{\text{weak}}/\sigma_{\text{ann}}$ or larger WIMP mass (probed by LHC!)
- generation of the seeds of (extra)galactic magnetic fields:
  $\rightarrow$ possible within the standard model again
- modified gravitational wave background:
  observable with pulsar timing and LISA