What Can Solar Neutrinos Tell Us About the Solar Radiative Zone?

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Outline

• Neutrino Oscillations and Fluctuations
  • Neutrino Oscillations Measured
  • Constraining Solar-Medium Fluctuations
• Observable Fluctuations from Magnetic Fields?
  • Helioseismology and Magnetic Fields
  • The Alfvén-wave/g-mode Resonance.
• Conclusions
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The Sun

- Dedicated satellite measurements are allowing many of the Sun’s properties to be understood.
  - Most of these are restricted to measurements of the solar surface.
The Solar Interior

- Solar structure is inferred by modelling, based on bulk properties.
  - $T_{\text{surf}}$, $L_\odot$, $R_\odot$ ...
- Now supplemented by two direct probes:
  - Helioseismology
  - Solar neutrinos
Solar neutrino deficit is now understood as a consequence of neutrinos oscillating while en route from the solar core.
After Seasoning with Salt

Allowed oscillation parameters after SNO salt
Terrestrial Evidence

- The evidence for neutrino oscillations is also now supported by direct terrestrial experiments, which measure disappearance of anti-$\nu_e$'s from reactors en route to a distant detector.
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What Can We Learn About the Sun?

- The success of the solar model prediction constrains thermal properties deep within the solar core.
  - The $^8$B flux is sensitive to the temperature at the production point.
  - The efficiency of oscillations also depends on the environment encountered en route.

![Diagram of MSW Resonance Radius in the Sun](image)
Sensitivity to Solar Fluctuations

- Resonant neutrino oscillations can be sensitive to fluctuations in the solar environment.
  - Each neutrino is lucky to scatter even once on its way out.
  - Successive neutrinos ‘see’ different solar properties, and so can oscillate more or less efficiently.
  - Jump probability seen by neutrinos behaves as a random variable: (‘Generalized Park Formula’)

CB & Michaud

Loreti, Qian, Fuller & Balantekin

Nunokawa, Rossi, Semikoz & Valle
Only Fluctuations at Resonance Count

Oscillations are only sensitive to fluctuations which satisfy two criteria at resonance:

- Their correlation length must be comparable to the neutrino oscillation length.
- Their amplitude must be at least a few percent of the background values.

No known sources of solar oscillations seem to have the required properties.

CB, Bamert & Michaud
Better knowledge of neutrino oscillation parameters allows better sensitivity to solar properties.

- Fluctuations generically imply both a reduced transition rate and a distortion of the energy spectrum.
Pre-Salt Sensitivities

Better knowledge of neutrino oscillation parameters allows better sensitivity to solar properties.

- Fluctuations generically imply both a reduced transition rate and a distortion of the energy spectrum.

\[
\tan^2 \theta = 0.43 \\
\Delta m^2 = 6.9 \times 10^{-5} \text{ eV}^2
\]
Sensitivity vs Experiment Type

Relative Chi-Squared vs Noise Amplitude
Sensitivity of Oscillation Parameters

Fits to oscillation parameters with and without noise.
(Black lines indicate same results without KamLAND)
Sensitivity of Oscillation Parameters

with and without SNO salt

Fits to oscillation parameters with and without noise. (Black lines indicate same results without KamLAND)
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Helioseismology

- The solar interior supports numerous kinds of oscillations, whose motion on the solar surface can be detected.
  - Comparison of frequency and observed wavepattern with solar model predictions constrains the properties of the model as a function of depth below the surface.
Two Kinds of Waves

- Several kinds of waves are possible.
- For $p$-modes pressure is the restoring force.
  - Higher frequency, closer to solar surface.
- For $g$-modes buoyancy is the restoring force.
  - Lower frequency, confined to radiative zone.
Physical Properties vs Depth

- Although helioseismology constrains deviations from solar models at better than the percent level, it cannot resolve fluctuations with small spatial sizes.
  - Must deconvolve spherical harmonics given only half of the Sun and experimental noise.
Solar magnetic fields have long been believed to be generated by a solar dynamo, in which the turbulent inner motion generates the magnetic fields we see.

- Longstanding problem to understand this dynamo in detail.
New Progress from Helioseismology

- Helioseismology now allows a more detailed 3-dimensional inference of the motion of the solar interior in the Convective Zone.
  - This new information is helping to pin down the nature of the solar dynamo.
  - Also new information from other stars, eg: the existence of large magnetic fields around convective brown dwarfs.
The layer at the base of the Convective Zone is now believed to play an important role in the dynamo, due to the large shear stresses there.

- The RZ rotates rigidly while the CZ rotates differentially.
- RZ toroidal fields may be as large as $10^4 - 10^5$ Gauss just below the CZ.
Constraints on RZ Magnetic Fields

- RZ fields can persist for the age of the Sun.
  - High RZ conductivity reduces magnetic diffusion.
- RZ fields must be less than \( \sim 10^7 \) Gauss:
  - Magnetic pressure \( B^2/8\pi \) would otherwise cause observable distortions to the solar oblateness.
  - Magnetic pressure in core would change densities and temperatures inconsistently with observed solar neutrino production and helioseismology.
  - Magnetic flux tubes would too rapidly rise out of the RZ into the CZ.
- Not clear how large RZ fields would arise in early Sun.
  - Fields ejected before RZ forms; subsequent generation < few G.

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Linear Magneto-Gravity Waves

- Can re-examine helioseismic waves in the RZ and see if magnetic fields can make a difference.
- Restrict to plasma properties deep in RZ.
- Restrict to rectangular geometry where $B_0$ is approximately constant and is perpendicular to $\nabla \rho$ and $g$.
  - Best near solar equator, not too close to solar centre.
  - Requires times much shorter than rotation period.

\[
\begin{align*}
\frac{\partial \rho'}{\partial t} + (v \cdot \nabla) \rho + \rho u &= 0, \\
\rho_0 \frac{\partial v}{\partial t} + \nabla p' - g \rho' - \frac{1}{4\pi} \left[ \text{rot} \ B' \times B_0 \right] &= 0, \\
\frac{\partial p'}{\partial t} + (v \cdot \nabla) p_0 + \gamma p_0 u &= 0, \\
\frac{\partial B'}{\partial t} &= \text{rot} \left( v \times B_0 \right),
\end{align*}
\]

\[
\frac{\partial^2 v}{\partial t^2} = \nabla (g \cdot v) + (\gamma - 1) g u + c_s^2 \nabla u - v_A^2 \frac{\partial}{\partial t} \left[ \frac{B_0}{B_0} \times \text{rot} \ b \right]
\]
Consider magnetic fields perpendicular to neutrino path and to pressure and density gradients. (e.g.: toroidal, poloidal)
Rectangular coordinates can approximate such magnetic fields far from the solar center, for time scales short compared with the solar rotation period.
Approximate MG Equations

\[ b_x(z) = \frac{ik_x}{k_x^2} \frac{db_x(z)}{dz} , \]
\[ b_y(z) = \frac{ik_y}{k_y^2} \frac{db_y(z)}{dz} , \]
\[ v_x(z) = \frac{\omega}{k_x} b_x(z) , \]
\[ v_y(z) = \frac{\omega}{k_y} b_y(z) , \]
\[ v_x(z) = \frac{ik_x \omega}{k_x^2} \left[ -\frac{g b_x(z)}{k_x c_s^2} + \left( 1 + \frac{\omega^2}{k_x^2 c_s^2} \right) \frac{1}{k_x} \frac{db_x(z)}{dz} \right] \]
\[ v_y(z) = \frac{ik_y \omega}{k_y^2} \left[ -\frac{g b_y(z)}{k_y c_s^2} + \left( 1 + \frac{\omega^2}{k_y^2 c_s^2} \right) \frac{1}{k_y} \frac{db_y(z)}{dz} \right] \]
\[ p'(z) = i \frac{\gamma \omega^2}{k_x c_s^2 k_x} \frac{db_x(z)}{dz} , \]
\[ \rho'(z) = \frac{(+i)}{k_x H} \left( \frac{\gamma - 1}{\gamma} b_x(z) + \frac{\omega^2 H}{k_x c_s^2} \frac{db_y(z)}{dz} \right) , \]
\[ N^2(z) = g(z) \left( \frac{1}{\gamma p_0} \frac{dp_0}{dz} - \frac{1}{p_0} \frac{dp_0}{dz} \right) \]

- Reduce coupled system to a single eigenvalue equation for \( b_z \).
Alfvén Waves

- Magnetic fields introduce new modes: Alfvén waves.
  - Waves can propagate along magnetic field lines.

\[ v_A^2 = \frac{B^2}{4\pi \rho} \]

\[ \omega = v_A k_{||} \]
Alfvén/g-mode Resonance

- The frequency of Alfvén modes crosses the frequency of g-modes at specific radii, causing resonances.
- Can channel energy from g-modes along magnetic field lines.
Surprisingly Large Magnetic Effects

- Magneto-Gravity wave profiles show resonant spikes at specific radii.
  - Resonant position depends on mode number.
  - Resonant position depends on magnetic field strength.
  - Resonant width can be comparatively narrow.
Normal Mode Frequencies

- Real and imaginary parts of the Magneto-Gravity dispersion relation.
  - Frequency cannot be as small as $1/27$ day $= 4 \times 10^{-7}$ Hz.
Fluctuations on $\nu$ Oscillation Scales.

- For magnetic fields of order 10 kG, these density spikes can alter neutrino oscillations.
  - They arise where the neutrinos oscillate.
  - Their spacing can be comparable with the $\nu$ oscillation length.
- What is their amplitude??
Observable Changes to Oscillations?

- If these modes are large enough and the magnetic fields are the right size, then they appear as density fluctuations as viewed along a neutrino trajectory. This can lead to observable dilution of neutrino oscillations given sufficient amplitude.
  - Most efficient near solar equatorial plane: seasonal effect?
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Summary

• Solar neutrino physics is entering a mature phase where neutrinos can be used to probe the deep solar interior.
  • Probes both the neutrino production point and the intervening neutrino oscillation history.

• Solar fluctuations at the position of the resonant neutrino oscillations can be observable if their amplitude is a few percent and their correlation length is a few hundred km.
  • Sensitive to scales much shorter than helioseismology.

• Under favourable circumstances, radiative zone magnetic fields may produce such fluctuations in the solar density due to a resonance between Alfvén waves and helioseismic g-modes.