Is Isotropic turbulence relevant in the atmosphere?

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Abstract:

The problem of turbulence is ubiquitous in the earth sciences, astrophysics and elsewhere. Virtually the only theoretical paradigm that has been seriously considered is strongly isotropic in the sense that scaling exponents are the same in all directions so that any remaining anisotropy is “trivial”. Using 235 state of the art drop sondes data sets of the horizontal wind at ≈5m resolution in the vertical, we show that the atmosphere is apparently outside the scope of these isotropic frameworks. It suggests that anisotropy may frequently be strong requiring different scaling exponents in the horizontal and vertical directions.

If we include intermittency, Kolmorogov’s [Kolmogorov, 1941] landmark proposal that fully developed turbulence has an “inertial subrange” with isotropic energy spectrum $E(k) \approx k^{-5/3}$ has apparently been spectacularly confirmed in both the horizontal direction and in the time domain ($k$ is a wavenumber). For gradients over a horizontal distance $\Delta x$ this implies

$$\langle |\Delta v(\Delta x)| \rangle \sim \Delta x^{H_h}$$

($H_h=1/3$ corresponds to $\beta=5/3$; “<.$$” indicates ensemble averaging).

Remarkably, $H_v$ for gradients over vertical distances $\Delta z$ ($\langle |\Delta v(\Delta z)| \rangle \sim \Delta z^{H_v}$) has not been seriously investigated. Using state-of-the-art drop sonde data of horizontal wind, we find that from scales of 5 m to >10 km from the surface layer through to the top of the troposphere, $H_v$ is close to (or larger) than the Bolgiano[Bolgiano, 1959]-Obukhov[Obukhov, 1959] value 3/5. $H_v>H_h$ implies that a) the atmosphere becomes progressively less stratified at smaller scales although in a scaling
way [Schertzer and Lovejoy, 1985a]; b) that at most a single (roughly) isotropic “sphero-scale”
exists (often [Lilley, et al., 2004; Lovejoy, et al., 2004] in the range 1-100 cm).

Kolmorogov’s theory is based on two key assumptions: a) that there exists an “equilibrium”
range where the turbulence is isotropic depending only on the energy flux $\epsilon$, and the viscosity $\nu$, b)
that within the equilibrium range, an inertial subrange exists where only the scale-by-scale transport
of energy is important; this is the $k^{5/3}$ regime. The main reason for supposing that a) is valid in the
atmosphere is that in turbulence, structures at a given scale are mostly coupled with structures at
neighbouring scales so that the effects of large scale boundary conditions are progressively
“forgotten” at small scales. Classically this tendency to “return to isotropy” [Rotta, 1951] has been
modeled using second order closure techniques; however even within this framework, when
buoyancy forces are included, they are found to be relatively large [Moeng and Wyngaard, 1986], just
as in laboratory flows it is found that even small buoyancy forces readily destroy isotropy [Van Atta,
1991]. Even recent theoretical advances [Arad, et al., 1998; Arad, et al., 1999] assume a priori that
fluctuation statistics follow the form $\langle |\Delta v(\Delta r)| \rangle = \Theta (\hat{\Delta r}) |\Delta r|^H$ where $\hat{\Delta r}$ is the angle, $|\Delta r|$ the
length of the separation vector $\Delta r$, i.e. they assume that $H_\omega=H_v$ and introduce the “trivial
anisotropy” function $\Theta (\hat{\Delta r})$; indeed they introduce a hierarchy of such terms each with different
$H$’s and $\Theta$’s. Since the theory ignores buoyancy, when it was checked in the atmosphere, the data
were restricted to the horizontal [Kurien, et al., 2000]. Indeed, virtually all empirical surface layer
atmospheric tests of isotropy (i.e. those with the best quality data) simply assume that $H_\omega=H_v=1/3$
and test the anisotropy at unique scales. It is even common to study the spatial anisotropy of scalars
by using single point time series of gradients, converting time to space with “Taylor’s hypothesis”
of frozen turbulence, and then using the skewness to determine the forward/backward trivial
anisotropy [Sreenivasan, 1991]. Even in the analysis of laboratory (Rayleigh-Bénard) convection
where there is a debate about whether $H=1/3$ or 3/5, isotropy (i.e. $H_h=H_v$) is still assumed and one
studies time series at single points[Ashkenazi and Steinberg, 1999; Shang and Xia, 2001].

Gravity however, acts at all scales through buoyancy effects, and it is precisely buoyancy effects
which lead to the hypothesis of a central role for the buoyancy variance flux[Bolgiano,
1959],[Obukhov, 1959]. The “hybrid” 23/9D anisotropic scaling model[Schertzer and Lovejoy,
1985a; 1985b] postulates that the energy flux dominates in the horizontal while the buoyancy
variance flux dominates in the vertical so that $H_h=1/3$ but $H_v=3/5$.

Most of our knowledge of the vertical structure of the atmosphere comes from radiosonde
balloons designed for synoptic forecasting rather than research; they typically have resolutions of
the order 150-200 m. In addition to their low resolutions, balloons suffer from swaying payloads
and disturbances on ascent caused by the balloon’s wake. In spite of these difficulties,
experimentalists largely interpret the vertical spectrum in terms of quasi-linear gravity waves with
exponent $H_v=1$ (but with $H_h\approx1/3$; see e.g.[Allen and Vincent, 1995; Dewan, 1997; Fritts, et al.,
1988; Gardner, 1994]). This follows from dimensional analysis if the layers are stable and
homogeneous with well-defined Brunt-Väisälä frequencies, see ref.[Lovejoy, et al., 2006] for a
critique. In comparison, the older Lumley-Shur[Lumley, 1964; Shur, 1962] model predicts an
isotropic $H=1$ regime. In order to test the Kolmogorov law in the vertical, we used state-of-the-art
drop sonde data from the NOAA Winter Storms 04 experiment over the Pacific Ocean, where 261
sondes were dropped from roughly 13 km altitudes. These GPS sondes had vertical resolutions of $\approx$
5 m, temporal resolutions of 0.5 s, horizontal velocity resolutions of $\approx 0.1$ m/s and temperature
resolutions of $\approx 0.1$ K [Hock and Franklin, 1999]. While the full analysis of the 2004 experiment is
described in ref[Hovde, et al., 2006b], we concentrate here on analysis of the key horizontal
velocities.

Fig. 1 shows the composite analysis of the most complete 235 sondes; of these near complete
data sets, outages were most frequent at the higher altitudes. For each sonde, the mean absolute
shears $\Delta v(\Delta z) = \left| v(z_i) - v(z_j) \right|$ ($v$ is the horizontal velocity vector) were calculated using all pairs of points with $\Delta z = |z_i - z_j|$ in logarithmically spaced intervals, and for all $z_i, z_j < z_i$ where $z_i$ is the indicated altitude threshold. This method is particularly effective since while there are $\approx 1400$ data points per sonde (at 2 Hz), there are many more pairs of points (roughly $10^6$); the method also overcomes the irregular vertical spacing of the data without requiring potentially problematic interpolations.

Three features of fig.1 are particularly striking: a) the overall scaling – even for the thickest layers spanning the entire troposphere – is excellent; the standard errors in the slope ($H$) estimates are $\pm 1\%$; b) the slopes at the lower levels (which are not too affected by the ever present strong jet streams) is very close to the BO value 3/5, but increases at higher at altitudes, c) there is no evidence for any $H=1/3$ behaviour, even in the lowest layer ($<158$ m), and at the smallest scales (5 m). However, since this figure pools the data from all the sondes, the result might be an artifact of mixing data from profiles some of which might have Kolmogorov scaling. Fig. 2 shows histograms, altitude by altitude giving the distribution of $H$ values. In this case, the layers spaced linearly, and regressions are made over layers 1km thick with $(z_i + z_j)/2$ in the range $z_i-1$ km $<z< z_i$, for 5 m$<\Delta z<$1 km. We note that while the distributions are generally bell-shaped that of the 235 sondes, only a single one at the lowest 1km level has $H \approx 1/3$. In order to quantitatively characterize the mean and spread of these values, we refer to fig. 3 (top left) which gives the one standard deviation spread of values around the mean. One can see that the Kolmorogov $H=1/3$ value is systematically 2-4 standard deviations below the mean. We also see that the slight difference in analysis method tends to give a small but systematic difference of $H$.

We now examine several factors which may affect the result. First, the structure function - although simple to apply - is limited to series with $0<H<1$; we therefore also applied a version of the Detrended Fluctuation Analysis[Kantelhart, et al., 2002] (adapted to irregular data spacing)
which systematically removes linear trends, effectively redefining the fluctuation $\Delta v$ as the
difference between $v$ and a linear estimate; this change only made a small change in the $H$ estimates
(it increased them slightly). Second, we may wonder how much the estimates are affected by
intermittency; although a priori this effect will be small for the first order moments. To
quantitatively characterize it, we calculated the $q^{th}$ order structure function exponent $\xi(q)$:

$$\left< |\Delta v|^q \right> = \Delta v^{\xi(q)}. $$

Increasing $q$ yields statistics more and more sensitive to (rare) large fluctuations; fig. 3 (top) shows the $H$ values cited above (= $\xi(1)$) and the spectral exponent $\beta = 1+ \xi(2)$. To characterize $\xi(q)$, we fit it to the following “universal multifractal[Schertzer and Lovejoy, 1987]”
parametric form: $\xi(q) = Hq - K(q), \quad K(q) = \frac{C_1}{\alpha - 1} (q^\alpha - q)$ where $C_1$ is the codimension of the mean
fluctuation and $\alpha$ is the Levy index characterizing the degree of multifractality. $C_1$ quantifies the
effect of intermittency on the mean; if this was large enough it could perhaps explain the large $H$
values. Fig. 3 (bottom left) shows that $C_1$ is quite small (much less than $H$), so that we cannot
explain the deviations from Kolmogorov’s law due to intermittency effects. The $\alpha$ values (fig. 3
bottom right) show that the effects of intermittency increase rapidly with $q$ ($\alpha$ is near its theoretical
maximum, 2).
Fig. 1: Rms fits to the sonde mean absolute vertical shears of horizontal wind for layers of thickness increasing logarithmically. The reference lines have slopes $H=1/3$ (Kolmogorov), $H=3/5$ (Bolgiano-Obukhov), $H=1$ (gravity waves). The rms $H$ estimates are given next to the lines. The data for each level are offset by one order of magnitude for clarity, units m/s.
Fig. 2: Histogrammes of $H$. Sonde by sonde, layer by layer frequency the figure displays the distribution of $H$ values calculated from the modulus of the vector velocity differences (offset in the vertical for clarity, successive histogrammes are for increasing 1km thick layers). For each histogramme, $H$ is estimated over all points whose mean altitude is between the indicated layer altitude and 1km below it, the fits are over the scales 5-1000m. The arrows show the theoretical exponents for reference.
Fig. 3: Scaling exponents as functions of altitude. **Top Left:** The means and standard deviations of the $H$ values calculated from the moduli of the vector differences in horizontal winds. The blue curve is from the $H$ values in fig. 1, i.e. over all pairs of points below the altitude indicated, estimated over the entire range of scales available (i.e. up to 12.6 km at the highest altitudes). The points are fits from individual sondes, as indicated in fig. 2. The error bars indicate the sonde to sonde variability (235 sondes were used). **Top right:** Same but for the corresponding spectral exponents $\beta$, (nonintermittent) Kolmogorov theory yields $\beta=5/3$, Bolgiano Obukhov, $\beta=11/5$. **Bottom Left:** The $C_1$ values corresponding to the north-south components corresponding to the top. **Bottom Right:** The corresponding $\alpha$ values.

Using 235 state of the art drop sondes over the northern Pacific Ocean, we have shown that there is no evidence that the Kolmogorov law – or its intermittent generalizations – hold anywhere in the troposphere from scales 5 m and up, for any layers, even those within 158 m of the surface. This is almost certainly a consequence of the fact that buoyancy forces are always important. Note that since $H>H_s$, structures will indeed be more isotropic at small scales, even perhaps being exactly isotropic at a unique “sphero-scale”. However this has nothing to do with a “return to isotropy” or independence of the turbulence with the large scale forcing; rather it is a “cross-over” phenomenon, i.e. a consequence of two power laws “crossing” at a unique scale.

While these results are consistent with previous radiosonde studies (see for a review ref.[Lilley, et al., 2005], for new results[Hovde, et al., 2006a]), they are precise enough to be much more
conclusive. Indeed, their high precision has brought to the fore a systematic tendency for the $H$ values to increase from the near surface Bolgiano-Obukhov value $3/5$ to values closer to 0.77 in higher layers subject to large (jet) shears. While the exact explanation for this increase is unclear at present, it should be recalled that like the Kolmogorov law, the Bolgiano-Obukhov law presupposes spatial statistical homogeneity, which is violated by the strongly altitude dependent jets. In this respect it is significant that an analysis of data over land and in the lower 4 km (without strong shears) using aerosols as passive tracers[Lilley, et al., 2004] (detected by high resolution lidars), found $H_r \approx 0.60 \pm 0.03$ (compared to $H_r \approx 0.33 \pm 0.02$, close to the standard Corrsin-Obukhov value $1/3$).

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References


**Figure Captions:**

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