Atmospheric complexity or scale by scale simplicity?

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Abstract:

Is the numerical integration of nonlinear partial differential equations the only way to tackle atmospheric complexity. Or do cascade dynamics repeating scale after scale lead to simplicity? Using 1000 orbits of Tropical Rainfall Measuring Mission (TRMM) satellite radiances from 10 bands representative of the short wave (visible, infra red) and long wave (passive microwave) regions and 8.8 to 20,000 km in scale, we find that the radiance gradients follow the predictions of cascade theories to within about ±0.5%, ±1.25% for the short and long waves respectively and with outer scales varying between ≈ 5,000 to ≈ 28,000 km depending on the band. Since the radiances and dynamics are strongly coupled, we conclude that weather can be accurately modeled as a cascade process.

In 1922, Lewis Fry Richardson published the now celebrated book “Weather forecasting by numerical process”(1) in which he daringly proposed that the weather could be forecast by brute force numerical integration of coupled nonlinear partial differential equations (PDE’s). But the father of numerical weather predication (NWP) was Janus-faced: his book contains a famous footnote in which he proposed that the complex nonlinear atmospheric dynamics cascaded scale after scale from planetary down to small viscous scales. Shortly afterwards (2) he suggested that atmospheric particle trajectories might be Wierstrasse-like functions (fractals) with simple (but nonclassical) scale by scale regularity. Richardson apparently believed that messy complexity ought to give way to scale by scale simplicity: he is often considered the grandfather of modern cascade models.
Today, numerical forecasting is a daily reality; but what about the dream of scale by scale simplicity? Explicit cascade models were developed in the 1960’s and 70’ (e.g. (3), (4), (5)) and by the 1980’s it was realized that they were the generic multifractal process. Subsequent developments have shown their great generality spawning applications throughout physics and the geosciences. In particular, while today there is a general consensus that at least over some scale range the atmosphere is multifractal, there have not yet been planetary scale investigations of the cascade hypothesis. One of the reasons is that the dynamically most important fields must be measured \textit{in situ} and this introduces numerous difficulties of interpretation (for example both (sparse) networks and aircraft trajectories can themselves be fractal (6), (7)). Consequently it is advantageous to use remotely sensed radiances: the largest relevant study (8) used nearly one thousand 256X256 pixel “scenes” of satellite visible and Infra red radiances over the range 2.2 to 280 km. While the fields accurately displayed cascade statistics, the largest scales - including the key outer scale of the variability - was only indirectly estimated by extrapolation beyond the measured range.

Although the scaling study (8) had a hundred times the data content of the largest in situ turbulence experiment - it was small by today’s standards. In this paper, we use about one thousand orbits of visible, infra red (IR) and passive microwave data (10 bands in all) from the Tropical Rainfall Monitoring Mission (TRMM) satellite to extend these analyses to global scales. Each orbit comprises about the same amount of data as the entire previous study.

The TRMM satellite was launched on November 27, 1997, in an orbit between ±38° latitude at 350km altitude (period of 91 minutes). We analyse data from the Visible and Infrared Scanner (VIRS) (9) and the Thermal Microwave Imager (TMI) (10). VIRS has five separate bands, ranging from the visible to thermal infrared (Table 1). The nominal resolutions were 2.2 km, with a 720 km swath width. TMI has nine microwave bands (four of which are dual polarization) with swath width 760 km (Table 2). The nominal resolution at the highest frequency (85.5 GHz, ≈ 3.5 mm) was 4.2X6.8 km (cross-track X along track) with the other bands having lower resolutions
decreasing to 36 × 60 km at (10.6 GHz ≈ 3.0 cm) with 13.9 km between successive scans. Since
the scaling properties of the horizontal and vertical polarizations were very similar, we only
analyzed the five vertically polarized bands indicated in Table 2.

Although analyses were performed on orbits 538 through 1538 (in 1998), each band has differing
fractions of missing data (10-15% were discarded). This roughly two month period was chosen
because it was about 2 – 4 times the typical lifetime of global scale structures (the “synoptic
maximum”): analysis of first half of the data indeed gave nearly identical results. In comparison,
the average return time for the satellite was about 2 days.

If the atmospheric dynamics are controlled by scale invariant turbulent cascades of various
(scale by scale) conserved fluxes then the gradients of the radiances (ε) are the result of a pure
multiplicative cascade and the normalized statistical moments (M_q) obey the generic multiscaling
relation:

\[ M_q = \left( \frac{\lambda}{\lambda_{\text{eff}}} \right)^{K(q)} \]

\[ \lambda = \frac{L_{\text{earth}}}{L}; \quad \lambda_{\text{eff}} = \frac{L_{\text{earth}}}{L_{\text{eff}}}; \quad M_q = \frac{\langle \varepsilon_1^q \rangle}{\langle \varepsilon_1 \rangle^q} \]

where \( L_{\text{eff}} \) is the effective outer scale of the cascade and \( L \) is the resolution at which it is
measured/averaged. \( \langle \varepsilon_1 \rangle \) is the ensemble mean large scale (i.e. the climatological value). \( L_{\text{earth}} = 20,000 \) km is a reference scale conveniently taken the largest great circle distance on the earth and
scale ratio \( \lambda_{\text{eff}} \) is determined empirically.

In Figure 1 we show the results on the 5 VIRS bands. For reference, we have plotted the
regressions in which the slope \( K(q) \) was fitted to each line, but the intercept forced to go through the
common point \( \lambda = \lambda_{\text{eff}} \). We see that to high accuracy out to near planetary scales, the only
significant difference between the gradient statistics for different wavelengths is the outer scale.
From Table 1 we can see that \( L_{\text{eff}} \) is in the range of about 11000 – 28000 km indicating that over the
range where the straight lines approximate the data well, that the variability of weak and strong
structures (large and small $q$) is the same as that produced by a multiplicative cascade. From the figures we see that the very large scales depart a little from the pure scaling only for scales $> 5000$ km (far left). To further quantify the differences between wavelengths we must compare the slopes (the $K(q)$ functions). A simple way to do this which is valid near the mean ($q = 1$) is to use the parameter $C_1 = K'(1)$ called “the codimension of the mean”; see Table 1. $C_1$ quantifies the sparseness of the field values which give the dominant contributions to the mean (for a full characterization universal multifractals can be used, e.g. (11)).

To understand the results in Table 1, we note that the VIRS bands 1, 2 are essentially reflected sunlight (with very little emission and absorption) so that for thin clouds, the signal comes from variations in the surface albedo (influenced by the topography and other factors), while for thicker clouds it comes from nearer the cloud top via (multiple) geometric and Mie scattering. As the wavelength increases into the thermal IR, the radiances are increasingly due to black body emission and absorption with very little multiple scatter. Whereas at the visible wavelengths we would expect the signal to be influenced by the statistics of cloud liquid water density ($C_1 \approx 0.07$, (12), (13)) – itself close to those of passive turbulent scalars ($C_1 \approx 0.04$ see the review in (14)) – for the thermal IR wavelengths it would rather be dominated by the statistics of temperature variations ($C_1 \approx 0.10$, (14)) – themselves also close to those of passive scalars. Elsewhere we quantify the shape of the $K(q)$ curves using universal multifractals showing that the $K(q)$ functions are close to those of previous visible and infra red studies performed at smaller scales (ground photography, SPOT, AVHRR and GMS satellites (15)).

In order to quantify the accuracy to which scaling is obeyed, we can determine the small deviations by estimating the mean absolute residuals:

$$
\Delta = \left| \log_{10} \left( M_q \right) - \log_{10} \left( \frac{\lambda}{\lambda_{\text{eff}}} \right)^{K[q]} \right|
$$

(2)
For each $q$, $\Delta$ is determined from the linear regression on Fig. 1; the slopes yield $K(q)$ and $\lambda_{eff}$ is determined from the intercept (fixed to be the same for all $q$). The overbar in equation (2) indicates averaging over the different $\lambda$ (at intervals of $10^{0.2}$) over the available range of scales up to 5000 km. For $0 \leq q \leq 2$ (corresponding to $>90\%$ of the data) we find that the scaling of the fluxes is within $\Delta = 0.015$. Defining the percentage deviation $\delta = 100 \times (10^\Delta - 1)$ this implies $\delta < \pm 3.5\%$. The mean $\bar{\delta}$ over the range $0 \leq q \leq 2$ is given in Table 1; it is in the range $\pm 0.35$ to $\pm 0.61\%$.

The analogous analyses for the TMI data are shown in fig. 2 with $\lambda_{eff}$, $\bar{\delta}$ given in Table 2. We see that $\bar{\delta}$ is a little larger than from the VIRS ($\pm 1.01\% - \pm 1.66\%$). At the same time, as the wavelength increases from TMI 8 ($\approx 3.5$ mm) to TMI 1 ($\approx 3.0$ cm), $C_1$ tends to increase from roughly the VIRS value ($\approx 0.10$) to 0.26. It is instructive to compare these values to those of the TRMM (near) surface radar reflectivity ($Z$; bottom line Table 2). We see that $Z$ has an extremely high $C_1$; it also has stronger variability with $L_{eff}$ somewhat larger than the size of the earth implying that due to interactions with other atmospheric fields even globally averaged $Z$’s have the same residual variability that they would have had if the cascade had reached 32,000 km.

To understand these results, recall that the thermal microwave radiation has contributions from surface reflectance, water vapour and cloud and rain. Since the particles are smaller than the wavelengths this is the Rayleigh regime and as the wavelength increases from $\approx 3.5$ mm to $\approx 3.0$ cm the emissivity/absorptivity due to cloud and precipitation decreases so that more and more of the signal originates in the lower reaches of clouds and underlying surface. Also, the ratio of absorption to scattering decreases so that at 3 cm multiple scattering can be important in raining regions. The overall result is that the horizontal gradients - which we have used to estimate the cascade fluxes - will increasingly reflect large internal liquid water gradients. We therefore expect the longer wavelengths to give flux statistics close to those of the (2.2 cm) radar reflectivity signal (which is proportional to the second moment of the particle volumes). This explanation is
consistent with the trend mentioned above for $C_1$ to increase sharply at the longest wavelengths towards the reflectivity value. The relative similarity of the TMI 1 band and $Z$ (and the other bands with the VIRS) is also supported by the fact that the outer scale is in the 5,000 – 7,000 km range for the longer wavelengths but is nearly 16,000 km – approaching the reflectivity outer scale – in the TMI 1.

It is paradoxical that in spite of growing quantities of atmospheric data that there is still no accepted picture of the scale by scale statistical properties of the atmosphere, yet the high accuracy ($\approx \pm 1\%$) with which we show the scaling to be respected makes it one of the most accurately obeyed atmospheric laws. Since the radiances are strongly coupled with the dynamics, it is hard to avoid the conclusion the latter are scaling over virtually the entire meteorologically relevant range.

So which Richardson is right? The father of NWP or the grandfather of cascades? The answer may be both. This is possible because cascade models are specifically designed to satisfy many of the basic symmetries of the nonlinear PDE’s especially the scaling itself but also the conservation of fluxes such as energy which are conserved by the nonlinear terms. Up until now, the scaling properties of the models have been primarily studied in the time domain (16), (17), (18), however models are now large enough so that their (possible) spatial cascade properties can be directly studied. Work in progress (by some of the authors) on a typical GCM (the Canadian GEM model) do indeed show cascade behaviour in the horizontal wind up to $\approx 10,000$ km, so that the models catch a glimpse of the first factor of $\approx 30$ of a cascade which might continue down to millimeter scales. This raises the possibility of systematic scale by scale inter-comparisons of empirical and simulated fields and could open up new vistas for model evaluation by identifying spurious length scales and biased exponents.

While our results have numerous technical implications which we cannot elaborate here, the implications for the Earth’s energy budget are worth mentioning. Currently, this is estimated from satellite radiances combined using detailed radiative transfer models (see e.g. (19)). Since the
estimates depend nonlinearly on the satellite radiances and since \( K(q) \neq 0 \), the budget will depend on the (subjective) satellite resolutions, an effect which is not currently taken into account. This neglect therefore has implications for climate change.

The history of science has shown that apparently complex phenomena usually end up giving way to simplicity, and that simplicity points the way to the future. In this case, the discovery that dynamics are accurately cascade processes opens up promising new (stochastic) ways of understanding, modeling and forecasting the atmosphere (20) that directly exploit the scale by scale simplicity allowing us to model the enormous range of scales found in the atmosphere.

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References


Figure Legends:

Figure 1: This shows the moments of the cascade fluxes associated with the radiances from VIRS bands 1 - 5 (a –e). $\lambda = 1$ corresponds to 20000 km. The black lines are the regressions through the common outer scales indicated in table 1.

Figure 2: This shows the moments of the cascade fluxes associated with the radiances from TMI bands 1, 3, 5, 6, 8 (a –e). $\lambda = 1$ corresponds to 20000 km. The black lines are the regressions through the common outer scales indicated in table 2.
## Tables:

**Table 1:** The characteristics of the five visible and infra red bands.

<table>
<thead>
<tr>
<th>Band</th>
<th>Wavelength</th>
<th>Resolution (km)</th>
<th>$\delta$ (%)</th>
<th>$C_j$</th>
<th>$L_{off}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIRS 1</td>
<td>0.630 µm</td>
<td>8.8</td>
<td>0.53</td>
<td>0.077</td>
<td>13800</td>
</tr>
<tr>
<td>VIRS 2</td>
<td>1.60 µm</td>
<td>8.8</td>
<td>0.61</td>
<td>0.079</td>
<td>25000</td>
</tr>
<tr>
<td>VIRS 3</td>
<td>3.75 µm</td>
<td>22.</td>
<td>0.35</td>
<td>0.065</td>
<td>28200</td>
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<tr>
<td>VIRS 4</td>
<td>10.8 µm</td>
<td>8.8</td>
<td>0.37</td>
<td>0.081</td>
<td>11200</td>
</tr>
<tr>
<td>VIRS 5</td>
<td>12.0 µm</td>
<td>8.8</td>
<td>0.36</td>
<td>0.084</td>
<td>12600</td>
</tr>
</tbody>
</table>

**Table 2:** The characteristics of the five TMI bands. All used vertical polarization.

<table>
<thead>
<tr>
<th>Band</th>
<th>Wavelength</th>
<th>Resolution (km)</th>
<th>$\delta$ (%)</th>
<th>$C_j$</th>
<th>$L_{off}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMI 1</td>
<td>3.0 cm (10.6 GHz)</td>
<td>111.4</td>
<td>1.01</td>
<td>0.255</td>
<td>15900</td>
</tr>
<tr>
<td>TMI 3</td>
<td>1.58 cm (19.35 GHz)</td>
<td>55.6</td>
<td>1.25</td>
<td>0.193</td>
<td>6900.</td>
</tr>
<tr>
<td>TMI 5</td>
<td>1.43 cm (22.235 GHz)</td>
<td>27.8</td>
<td>1.66</td>
<td>0.157</td>
<td>5000.</td>
</tr>
<tr>
<td>TMI 6</td>
<td>8.1 mm (37 GHz)</td>
<td>27.8</td>
<td>1.51</td>
<td>0.15</td>
<td>4400.</td>
</tr>
<tr>
<td>TMI 8</td>
<td>3.51 mm (85.5 GHz)</td>
<td>13.9</td>
<td>1.26</td>
<td>0.102</td>
<td>6300.</td>
</tr>
<tr>
<td>TRMM* Z</td>
<td>2.2 cm (13.2 GHz)</td>
<td>4.3</td>
<td>6.0*</td>
<td>0.63</td>
<td>32000</td>
</tr>
</tbody>
</table>

* $Z = $ radar reflectivity factor, from (21). The minimum detectable signal is twice the mean so that most of the deviations from scaling are at low $q$. 
Figures:

Figure 1 a:

Figure 1 b:

Figure 1 c:

Figure 1 d:

Figure 1 e:
Figure 2a

Figure 2b

Figure 2c

Figure 2d

Figure 2e