Atmospheric complexity or scale by scale simplicity?

S. Lovejoy*, D. Schertzer², V. Allaire¹, T. Bourgeois², S. King¹, J. Pinel¹, J. Stolle¹

¹Physics, McGill University, 3600 University st., Montreal, Que. H3A 2T8, lovejoy@physics.mcgill.ca, 1-514-398-6537
²CEREVE, Université Paris Est, France, Daniel.Schertzer@cereve.enpc.fr, 33.1.64.15.36.33
³Météo France, 1 Quai Branly, Paris 75005, France

Abstract:

Is the numerical integration of nonlinear partial differential equations the only way to tackle atmospheric complexity? Or do cascade dynamics repeating scale after scale lead to simplicity? Using 1000 orbits of TRMM satellite radiances from 11 bands in the short wave (visible, infra red) long wave (passive microwave) and radar regions and 8.8 to 20,000 km in scale, we find that the radiance gradients follow the predictions of cascade theories to within about ±0.5%, ±1.25%, ±5.9% for the short waves, long waves and reflectivities respectively and with outer scales varying between ≈ 5,000 to ≈ 32,000 km. Since the radiances and dynamics are strongly coupled, we conclude that weather can be accurately modeled as a cascade process.
1. Introduction

In 1922, Lewis Fry Richardson published the now celebrated book “Weather forecasting by numerical process” [Richardson, 1922] in which he daringly proposed that the weather could be forecast by brute force numerical integration of coupled nonlinear partial differential equations (PDE’s). But the father of numerical weather predication was Janus-faced: his book contains a famous phrase in which he proposed that the complex nonlinear atmospheric dynamics cascaded scale after scale from planetary down to small viscous scales. Shortly afterwards [Richardson, 1926], he suggested that atmospheric particle trajectories might be Wierstrasse-like functions (fractals) with simple (but nonclassical) scale by scale regularity. Richardson apparently believed that messy complexity ought to give way to scale by scale simplicity: he is often considered the grandfather of modern cascade models.

Today, numerical forecasting is a daily reality; but what about the dream of scale by scale simplicity embodied in cascades? For a long time after Richardson, cascades were inspirational and were regularly invoked in turbulence theories. However, it was not until the development of explicit multiplicative cascade models (starting in the 1960’s and 70’s (e.g. [Novikov and Stewart, 1964], [Yaglom, 1966], [Mandelbrot, 1974])) that empirically verifiable cascade predictions could go much beyond the determination of (non intermittent) spectral exponents and of the up scale or down scale cascade direction (see e.g. the classical papers [Boer and Shepherd, 1983], [Chen and Wiin-Nielsen, 1978] and the update [Strauss and Ditlevsen, 1999]).

By the 1980’s it was realized that multiplicative cascade models were the generic multifractal process. Subsequent developments have shown their great generality which
have spawned applications throughout physics and the geosciences. In particular, while
today there is a general consensus that at least over some scale range the atmosphere is
multifractal, there have not yet been planetary scale investigations of the precise
predictions of these explicit cascade models (eq. 1 below). One of the reasons is that the
dynamically most important fields must be measured in situ and this introduces numerous
difficulties of interpretation (for example both (sparse) networks and aircraft trajectories
can themselves be fractal [Lovejoy, et al., 1986], [Lovejoy, et al., 2004] and sonde
outages can be multifractal [Lovejoy, et al., 2008c]). Consequently it is advantageous to
use remotely sensed radiances: the largest relevant study [Lovejoy, 2001] used nearly one
thousand 256X256 pixel “scenes” of satellite visible and Infra red radiances over the
range 2.2 to 280 km. While the fields accurately displayed cascade statistics, the largest
scales - including the key outer scale of the variability - was only indirectly estimated by
extrapolation beyond the measured range. Up until now, these shortcomings have made it
possible to dismiss the idea that scaling might hold up to near planetary scales or over
wide ranges and to continue to pursue approaches incompatible with scaling.

Although the study [Lovejoy, 2001] had a hundred times the data content of the largest
in situ turbulence experiment - it was small by today’s standards. In this paper, we use
about one thousand orbits of visible, infra red (IR), passive and active microwave data
(11 bands in all) from the Tropical Rainfall Monitoring Mission (TRMM) satellite to
directly extend these analyses to 20,000 km. Because of this wide range and the fact that
each orbit comprises about the same amount of data as the entire previous study, this
paper provides the first near “empirical proof” of wide range, planetary scale cascade
scaling.
2. The data

We analyze data from the Visible and Infrared Scanner (VIRS) [Barnes, et al., 1999], the Thermal Microwave Imager (TMI) [Kummerow, et al., 1997] and the precipitation radar (PR) (11) from the TRMM satellite (launched on November 27, 1997, in an orbit between ±38° latitude at 350km altitude, period of 91 minutes). VIRS has five separate bands, ranging from the visible to thermal infrared (Table 1). The nominal resolutions were 2.2 km, with a 720 km swath width. TMI has nine microwave bands (four of which are dual polarization) with swath width 760 km (Table 2). The nominal resolution at the highest frequency (85.5 GHz ≈ 3.5 mm) was 4.2X6.8 km (cross-track X along track) with the other bands having lower resolutions decreasing to 36 X 60 km at (10.6 GHz ≈ 3.0 cm) with 13.9 km between successive scans. Since the scaling properties of the horizontal and vertical polarizations were very similar, we only analyzed the five vertically polarized bands indicated in Table 2. Finally, we have included analysis of the active (PR) sensor which is a 2.2 cm wavelength radar with resolution 4.3 km in the horizontal and 250 m in the vertical (only near surface reflectivities were considered).

Although analyses were performed on orbits 538 through 1538 (roughly January and February 1998), each band has differing fractions of missing data (4 -15% were discarded). This roughly two month period was chosen because it was about 2 – 4 times the typical lifetime of global scale structures (the “synoptic maximum”): analysis of first half of the data indeed gave nearly identical results.
3. The analysis

If atmospheric dynamics are controlled by scale invariant turbulent cascades of various (scale by scale) conserved fluxes $\phi$ then the fluctuations $\Delta I(\Delta x)$ in the radiances over a distance $\Delta x$ are related to the turbulent fluxes by a relation of the form $\Delta I(\Delta x) \approx \phi \Delta x^H$ (this is a generalization of the Kolmogorov law for velocity fluctuations). Without knowing $H$ - nor even the physical nature of the flux - we can use this to estimate the normalized (nondimensional) flux at the smallest resolution of our data: $\phi/\langle \phi \rangle = \Delta I/\langle \Delta I \rangle$ (where “$\langle \rangle$” indicates statistical averaging). In this case, $\Delta I(\Delta x)$ was estimated by absolute differences: $\Delta I(\Delta x) = |I(x + \Delta x) - I(x)|$ with $\Delta x$ the smallest reliable resolution and $x$ an along track coordinate, but other definitions of fluctuations could be used. This flux can then be degraded (by averaging) to a lower resolution $L$. If the fluxes are realizations of pure multiplicative cascades then the normalized statistical moments $M_q$ obey the generic multiscaling relation:

$$M_q = \left( \frac{\lambda}{\lambda_{eff}} \right)^{\kappa(q)} ; \quad \lambda = L_{earth} / L ; \quad \lambda_{eff} = L_{earth} / L_{eff}$$

where $M_q = \left( \langle \phi \lambda \rangle / \langle \phi_\lambda \rangle \right)^q$ and $L_{eff}$ is the effective outer scale of the cascade. $\langle \phi_\lambda \rangle$ is the ensemble mean large scale (i.e. the climatological value). $\lambda$ is a convenient scale ratio based on the largest great circle distance on the earth $L_{earth} = 20,000$ km and the scale ratio $\lambda/\lambda_{eff}$ is the overall ratio from the scale where the cascade started to the resolution scale $L$, it is determined empirically.
In Figure 1 we show the results on the 5 VIRS bands. For reference, we have plotted the regressions in which the slope $K(q)$ was fitted to each line, and the intercept forced to go through the common point $\lambda = \lambda_{eff}$. We see that to high accuracy out to near planetary scales, the only significant qualitative difference between the flux statistics for different wavelengths is the outer scale. From Table 1 we can see that $L_{eff}$ is in the range of about 11000 – 28000 km. This cascade “signature” of converging lines shows that the variability of weak and strong structures (large and small $q$) is the same as that produced by a multiplicative cascade. From the figures we see that the very large scales depart a little from the pure scaling only for scales > 5000 km (far left). To further quantify the differences between wavelengths we must compare the slopes (the $K(q)$ functions). A simple way to do this which is valid near the mean ($q = 1$) is to use the parameter $C_1 = K'(1)$ called “the codimension of the mean”; see Table 1. $C_1$ quantifies the sparseness of the field values which give the dominant contributions to the mean (for a full characterization universal multifractals can be used, e.g. [Schertzer and Lovejoy, 1987]).

To understand Table 1, we note that the VIRS bands 1, 2 are essentially reflected sunlight (with very little emission and absorption) so that for thin clouds, the signal comes from variations in the surface albedo (influenced by the topography and other factors), while for thicker clouds it comes from nearer the cloud top via (multiple) geometric and Mie scattering. As the wavelength increases into the thermal IR, the radiances are increasingly due to black body emission and absorption with very little multiple scatter. Whereas at the visible wavelengths we would expect the signal to be influenced by the statistics of cloud liquid water density ($C_1 \approx 0.07$, [Lovejoy and Schertzer, 1995], [Davis, et al., 1996]) – itself close to those of passive turbulent scalars
In order to quantify the accuracy to which scaling is obeyed, we can determine the small deviations by estimating the mean absolute residuals:

$$
\Delta = \left| \log_{10}(M_q) - K(q)\log_{10}\left(\frac{\lambda}{\lambda_{\text{eff}}}\right) \right|
$$

For each $q$, $\Delta$ is determined from the linear regression on Fig. 1; the slopes yield $K(q)$ and $\lambda_{\text{eff}}$ is determined from the intercept (fixed to be the same for all $q$). The overbar in equation (2) indicates averaging over the different $\lambda$ (at intervals of $10^{0.2}$) over the available range of scales up to 5000 km. For $0 \leq q \leq 2$ (corresponding to $>90\%$ of the data) we find that the scaling of the fluxes is within $\Delta = 0.015$. Defining the percentage deviation $\delta = 100x(10^\Delta - 1)$ this implies $\delta < \pm 3.5\%$. The mean $\delta$ over the range $0 \leq q \leq 2$ ($\bar{\delta}$) is given in Table 1; it is in the range $\pm 0.35$ to $\pm 0.61\%$.

The analogous analyses for the TMI data are shown in Fig. 2 with $\lambda_{\text{eff}}$, $\bar{\delta}$ given in Table 2. We see that $\bar{\delta}$ is a little larger than from the VIRS ($\pm 1.01\%$ - $\pm 1.66\%$). At the same time, as the wavelength increases from TMI 8 ($\approx 3.5$ mm) to TMI 1 ($\approx 3.0$ cm), $C_1$ tends to increase from roughly the VIRS value ($\approx 0.10$) to 0.26. It is instructive to compare these values to those of the TRMM (near) surface radar reflectivity ($Z$; Fig. 3 and
bottom line Table 2). We see that $Z$ has an extremely high $C_1$; it also has stronger variability with $L_{\text{eff}}$ somewhat larger than the size of the earth implying that due to interactions with other atmospheric fields even globally averaged $Z$’s have the same residual variability that they would have had if the cascade had reached 32,000 km. Although a curvature is visible for the low $q$ values, in [Lovejoy, et al., 2008a] this is quantitatively explained as an artifact of the insensitivity of the radar to low reflectivity values (the corresponding $C_1$ for the rain rate is $\approx 0.3$). The ability of the model to accurately predict not only the first order behaviour - but also the deviations from that behaviour – lends it further support.

To understand these results, recall that the thermal microwave radiation has contributions from surface reflectance, water vapour and cloud and rain. Since the particles are smaller than the wavelengths this is the Rayleigh regime and as the wavelength increases from $\approx 3.5$ mm to $\approx 3.0$ cm the emissivity/absorptivity due to cloud and precipitation decreases so that more and more of the signal originates in the lower reaches of clouds and underlying surface. Also, the ratio of absorption to scattering decreases so that at 3 cm multiple scattering can be important in raining regions. The overall result is that the horizontal gradients - which we have used to estimate the cascade fluxes - will increasingly reflect large internal liquid water gradients. We therefore expect the longer wavelengths to give flux statistics close to those of the (2.2 cm) radar reflectivity signal (which is proportional to the second moment of the particle volumes). This explanation is consistent with the trend mentioned above for $C_1$ to increase sharply at the longest wavelengths towards the reflectivity value. The relative similarity of the TMI 1 band and $Z$ (and the other bands with the VIRS) is also supported by the fact that
the outer scale is in the 5,000 – 7,000 km range for the longer wavelengths but is nearly
16,000 km – approaching the reflectivity outer scale – in the TMI 1.
4. Conclusions

It is paradoxical that in spite of growing quantities of atmospheric data that there is still no accepted picture of the scale by scale statistical properties of the atmosphere, yet the high accuracy ($\approx \pm 1\%$) with which we show the cascade structure to be respected makes it one of the most accurately obeyed atmospheric laws. Since the radiances are strongly coupled with the dynamics, it is hard to avoid the conclusion that the latter are spatially scaling over virtually the entire meteorologically relevant range. Elsewhere but with important nuances, we show that this conclusion also holds for temporal scaling.

So which Richardson is right? The father of NWP or the grandfather of cascades? The answer may be both. This is possible because cascade models are specifically designed to satisfy many of the basic symmetries of the nonlinear PDE’s especially the scaling itself but also the scale by scale conservation of fluxes such as energy which are conserved by the nonlinear terms. Up until now, the scaling (but not directly cascade) properties of the models have been primarily studied in the time domain ([Syroka and Toumi, 2001], [Blender and Fraedrich, 2003], [Fraedrich and Blender, 2003], [Kiehl and Trenberth, 1997]), however models are now large enough so that their (possible) spatial cascade properties can be directly studied. Analysis on a typical GCM (the Canadian GEM model, [Stolle, et al., 2008]) do indeed show cascade behaviour in the horizontal wind up to $\approx 10,000$ km, so that the models catch a glimpse of the first factor of $\approx 30$ of a cascade which might continue down to millimeter scales. Conventional models therefore already implicitly use cascades; however there are also explicit space-time stochastic cascade models [Marsan, et al., 1996], [Schertzer and Lovejoy, 2004], [Lovejoy and Schertzer, 2008] which have the advantage of being able take into account arbitrarily
large ranges of scale and of being able to directly produce “ensemble” forecasts. Since modern ensemble forecast systems require assumptions about the stochastic structure of the atmosphere, our results have direct applications for conventional modeling (e.g. “stochastic parameterisations” [Palmer, 2001]). Since current earth radiation budgets estimates do not take the scaling into account, there will also be applications to the assessment of climate change and to remote sensing.

The history of science has shown that apparently complex phenomena usually end up giving way to simplicity, and that simplicity points the way to the future. In this case, the discovery that model dynamics are themselves accurately modeled by cascade processes opens up promising new (stochastic) ways of understanding, modeling and forecasting the atmosphere [Schertzer and Lovejoy, 2004] that directly exploit the scale by scale simplicity allowing us to model the enormous range of scales found in the atmosphere.
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References


Lovejoy, S., et al. (2008a), The remarkable wide range scaling of TRMM precipitation, 


Figure Legends:

**Figure 1:** This shows the moments $q = 0.2, 0.4\ldots 1.8, 2.0,$ of the cascade fluxes associated with the radiances from VIRS bands 1 - 5 (a –e), $\lambda = 1$ corresponds to 20000 km. With the exception of the $q < 0.5$ lines, the curves increase with $q$ monotonically from bottom to top. The blue lines are the regressions through the common outer scales indicated in table 1, for each $q$, the slopes are the estimates of $K(q)$.

**Figure 2:** Same as fig. 1 but radiances from TMI bands 1, 3, 5, 6, 8 (a –e). The blue lines are the regressions through the common outer scales indicated in table 2.

**Figure 3:** Same as fig. 1 but reflectivities from the precipitation radar (wavelength 2.2 cm), $q = 0.1, 0.2\ldots 1.9, 2.0$. $\lambda = 1$ corresponds to 20000 km. The black lines are the regressions through the common outer scale indicated in table 2 (adapted from Lovejoy et al 2008).
### Tables:

#### Table 1: The characteristics of the five visible and infrared bands.

<table>
<thead>
<tr>
<th>Band</th>
<th>Wavelength (µm)</th>
<th>Resolution (km)</th>
<th>$\bar{\delta}$ (%)</th>
<th>$C_i$</th>
<th>$L_{eff}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIRS 1</td>
<td>0.630</td>
<td>8.8</td>
<td>0.53</td>
<td>0.077</td>
<td>13800</td>
</tr>
<tr>
<td>VIRS 2</td>
<td>1.60</td>
<td>8.8</td>
<td>0.61</td>
<td>0.079</td>
<td>25000</td>
</tr>
<tr>
<td>VIRS 3</td>
<td>3.75</td>
<td>22.</td>
<td>0.35</td>
<td>0.065</td>
<td>28200</td>
</tr>
<tr>
<td>VIRS 4</td>
<td>10.8</td>
<td>8.8</td>
<td>0.37</td>
<td>0.081</td>
<td>11200</td>
</tr>
<tr>
<td>VIRS 5</td>
<td>12.0</td>
<td>8.8</td>
<td>0.36</td>
<td>0.084</td>
<td>12600</td>
</tr>
</tbody>
</table>

#### Table 2: The characteristics of the five TMI bands. All used vertical polarization.

<table>
<thead>
<tr>
<th>Band</th>
<th>Wavelength</th>
<th>Resolution (km)</th>
<th>$\bar{\delta}$ (%)</th>
<th>$C_i$</th>
<th>$L_{eff}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMI1</td>
<td>3.0 cm (10.6 GHz)</td>
<td>111.4</td>
<td>1.01</td>
<td>0.255</td>
<td>15900</td>
</tr>
<tr>
<td>TMI 3</td>
<td>1.58 cm (19.35 GHz)</td>
<td>55.6</td>
<td>1.25</td>
<td>0.193</td>
<td>6900.</td>
</tr>
<tr>
<td>TMI 5</td>
<td>1.43 cm (22.235 GHz)</td>
<td>27.8</td>
<td>1.66</td>
<td>0.157</td>
<td>5000.</td>
</tr>
<tr>
<td>TMI 6</td>
<td>8.1 mm (37 GHz)</td>
<td>27.8</td>
<td>1.51</td>
<td>0.15</td>
<td>4400.</td>
</tr>
<tr>
<td>TMI 8</td>
<td>3.51 mm (85.5 GHz)</td>
<td>13.9</td>
<td>1.26</td>
<td>0.102</td>
<td>6300.</td>
</tr>
<tr>
<td>TRMM* Z</td>
<td>2.2 cm (13.2 GHz)</td>
<td>4.3</td>
<td>5.9*</td>
<td>0.63</td>
<td>32000</td>
</tr>
</tbody>
</table>

* Z = radar reflectivity factor, from [Lovejoy, et al., 2008a]. The minimum detectable signal is twice the mean so that most of the deviations from scaling are at low $q$. 
Figure 3.