1 Stochastic and scaling climate sensitivities: Solar, volcanic and orbital forcings

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[1] Climate sensitivity ($\lambda$) is usually defined as a deterministic quantity relating climate forcings and responses. While this may be appropriate for evaluating the outputs of (deterministic) GCM’s it is problematic for estimating sensitivities from empirical data. We introduce a stochastic definition where it is only a statistical link between the forcing and response, an upper bound on the deterministic sensitivities. Over the range ~30 yrs to 100 kyr we estimate this $\lambda$ using temperature data from instruments, reanalyses, multiproxies and paleo-sources; the forcings include several solar, volcanic and orbital series. With the exception of the latter - we find that $\lambda$ is roughly a scaling function of resolution $\Delta t$: $\lambda \approx \Delta t^{0.6}$, with exponent $0 < H_s \approx < 0.7$. Since most have $H_s > 0$, the implied feedbacks must generally increase with scale and this may be difficult to achieve with existing GCM’s. Citation: Lovejoy, S., and D. Schertzer (2012), Stochastic and scaling climate sensitivities: Solar, volcanic and orbital forcings, Geophys. Res. Lett., 39, LXXXXX, doi:10.1029/2012GL051871.

24 1. Introduction

[2] Even if one accepts that orbital forcing is the “pace maker of the ice ages” [Hays et al., 1976], over the range $27 \approx 30$ yrs to $\approx 30$ kyr, there is no doubt that most of the variance in paleotemperature records is associated with the continuous spectral “background” [Lovejoy and Schertzer, 1986; Wunsch, 2003] (for a recent spectrum see Figure S1 in Text S1 in the auxiliary material).1 This strongly suggests that other internal and/or external mechanisms are needed to explain the multidecadal, multicentennial and multimillennial variability. The discussion of these issues has been strongly tinted by the development of GCM’s and their response to various external climate forcings. However, if the amplification factors are large - as they must be - then it will be hard to distinguish nominally external forcing paradigms from purely internal ones.2

[3] The usual approach to evaluating climate forcings is via the climate sensitivity ($\lambda$) defined as the equilibrium change in a quantity, (here the temperature) per unit of radiative forcing. Sensitivities ($\lambda$) are commonly estimated with the help of (deterministic) numerical models; the usual example being the doubling of CO$_2$. The change in conditions (compositional in this example) simultaneously leads to changes in the typical mean global temperature ($\Delta T$) and to the earth’s radiative equilibrium from which the radiative forcing ($\Delta R_F$) is determined by:

$$\Delta T = \lambda \Delta R_F$$

This definition of climate sensitivity is convenient for numerical experiments with strong anthropogenic forcings. In this case, the response is relatively regular (smooth) so that the estimate $\lambda = \Delta R_F(\Delta T)/\Delta T(\Delta T)$ is well defined, insensitive to $\Delta T$. However, for natural forcings, it has several shortcomings. First, GCM outputs fluctuate over a wide range of $\Delta t$ so that for very small time scales comparable to the model integration time steps - fluctuations $\Delta T(\Delta t)$ (and presumably) $\Delta R_F(\Delta t)$ typically have non-trivial scaling behaviours $\Delta T(\Delta t) \approx \Delta T^{H_T}$ and $\Delta R_F(\Delta t) \approx \Delta R_F^{H_R}$ implying $\lambda(\Delta t) \approx \Delta R_F^{H_R}$ with $H_s = H_T - H_R$ generally non-integer. Second, the usual definition of climate sensitivity is only valid if there is a causal link: the fluctuations $\Delta T$ and $\Delta R_F$ must have the same underlying cause such as a change in solar output. Strictly speaking, it therefore cannot be used empirically since in the real world there is only a single realization of climate. From the climate record, we can only measure correlations, not causality. In addition to the causality assumption, empirical estimates of $\lambda$ must rely on model outputs in order to estimate $\Delta R_F$ [e.g., Harvey, 1988; Claquin et al., 2003; Chylek and Lohmann, 2008; Ganopolski and Schneider von Deimling, 2008].

[4] As a consequence of these difficulties, $\lambda$ has not been systematically explored as a function scale and it mostly known from models - not empirically. We therefore give a new stochastic definition of climate sensitivity which allows us to empirically estimate it for any physical forcing process whose consequent radiative forcing can be determined.

2 The Scaling of Temperatures, CO$_2$ Concentrations and Solar, Volcanic and Orbital Forcings

[5] Before considering potential climate drivers, let us first recall the variation with time scale $\Delta T$ of temperature fluctuations $\Delta T$. For this purpose, it turns out that it is not sufficient to define the fluctuation as the absolute difference $\Delta T$ $t=1T(t+\Delta t)-T(t)$. Instead, we should use twice the absolute difference of the mean of the temperature between $t$ and $t+\Delta t/2$ and between $t+\Delta t/2$ and $t+\Delta t$. Technically, this corresponds to defining fluctuations using Haar wavelets rather than “poor man’s” wavelets. While the latter is adequate for fluctuations increasing with scale (i.e., $\Delta T \approx \Delta T^{H_T}$ with $H_T > 0$), on average, absolute differences cannot
Figure 1. The RMS Haar structure function for temperatures including daily resolution 20th Century Reanalysis (20CR) data. On the left top we show grid point scale ($2^9 \times 2^9$) daily scale fluctuations for both 75°N and globally averaged along with reference slope $\xi(2)/2 = -0.4 \approx H$ (20CR, 700 mb). On the lower left, we see at daily resolution, the corresponding globally averaged structure function. Also shown are the average of three in situ surface series as well as a multiproxy structure function (northern hemisphere). At the right we show both the GRIP (55 cm resolution, with calibration constant 0.5 K/mil) and the Vostok paleotemperature series. Also shown is the interfacial “window” See Lovejoy and Schertzer [2012b] for the figure and a full description of the data.

[9] The variation of the fluctuations with scale can be defined using their statistics; the “generalized” $q$th order structure function $S_q(\Delta t)$ is particularly convenient:

$$S_q(\Delta t) = \langle \Delta T^2 \rangle^{q/2}$$

where "<" indicates ensemble averaging. In a scaling regime, $S_q(\Delta t)$ is a power law: $S_q(\Delta t) \approx \Delta t^{\xi(q)}$, where the exponent $\xi(q) = qH - K(q)$ and $K(q)$ characterizes the scaling intermittency (satisfying $K(1) = 0$). Below, with the exception of the volcanic series (where $K(2) \approx 0.2$), $K(2)$ is small ($\approx 0.01 - 0.03$), so that the RMS variation $S_2(\Delta t)^{1/2}$ has the exponent $\xi(2)/2 \approx \xi(1) = H$. Note that when $q = 2$ (the classical structure function), we have the useful relation $\xi(2) = \beta - 1$ where $\beta$ is the spectral exponent defined by the spectral density $E(\omega) \approx \omega^{-\beta}$ where $\omega$ is the frequency.

[7] When $S_2(\Delta t)^{1/2}$ is estimated for various in situ, reanalysis, multiproxy and paleo temperatures, then one obtains Figure 1 (see Table S1 in Text S1). The key points to note are a) the three qualitatively different regimes: weather, low frequency weather and climate with RMS fluctuations respectively increasing, decreasing and increasing again with scale ($H_w > 0$, $H_{lw} < 0$, $H_c > 0$) and with transitions at $\tau_w \approx 5 - 10$ days and $\tau_c \approx 10 - 30$ yrs, b) the difference between the local and global fluctuations, with the former decreasing from $\approx 5$ K (10 days) to $\approx 0.6$ K at $\approx 25$ yrs, increasing to $\approx 5$ K at 50 kys c) the “glacial/interglacial window” corresponding to overall $\pm 3$ to $\pm 5$K variations over scales with half periods of 30 – 50 kys. This basic multiscaling regime picture is similar to that of Lovejoy and Schertzer [1984, 1986, Pelletier [1998], and Huybers and Curry [2006]. For comparison, we could note that unforced GCM’s (control runs) at grid scale resolution have $H_{lw} \approx -0.4$ and do not yield any climate regime; i.e., $\tau_c \to \infty$ [see Lovejoy and Schertzer, 2012b].

[8] The problem of climate forcing is thus to determine what forcings might end the (decreasing, $H < 0$) low frequency weather regime and cause the fluctuations to start to increasing again when $\Delta t > \tau_c$ (i.e., $H > 0$)? To answer this, let us consider various possible external drivers as functions of scale; these may be conveniently classified according to whether they are scaling or nonscaling. This is useful because nonscaling climate forcings - i.e., at well defined frequencies – would leave strong signatures in the form of breaks in the temperature (and other) scalings which are generally not observed over the range of time scales between $\tau_c \approx 10 - 30$ yrs and $\tau_k \approx 50 - 100$ kys. An important nonscaling driver is the narrow-band orbital forcings at scales somewhat shorter but close enough to the upper time scale $\tau_k$. Although this break may well be
Turning to the higher frequency continuous back

Figure 3 we see that this range is about 3

indicates its dominant time scales. One sees that the vari-

Loutre
determined from astronomical calculations [176] irradiance variations at the north pole (every June 15th)

Orbital Variations of 90° N mid-June

ground, an (apparently) attractive possibility is to invoke

can be converted into radiative forcings (Figure 2). While to

within a constant factor (Figure 3) this is very nearly the same

corresponding to orbital eccentricity variations, but

also than the lower frequency 400 kyr eccentricity varia-

tions whose signal is virtually absent in the paleoclimate

Figure 2 shows (S2(Δt))^{1/2} from the 8 year long series from the TIMS satellite; we see clearly the 27

(earth) day long solar “day” followed by a low frequency rise. To go further requires proxy based “reconstructions”,

Figure 2 shows S2(Δt)\frac{1}{2} from several of these using sunspots and^{10}Be records. The earliest [Lean, 2000] used a two

component model, one of which had an 11 year cycle based on the recorded sunspots back to 1610, the other was a
“background”. Combining the two results leads to an annual series featuring an overall 0.21% variation in the background since the 17th century “Maunder Minimum”. Figure 2 shows that this reconstruction actually meshes quite nicely with the TIMS data with exponent $\xi(2)/2 \approx H_{RF} \approx 0.4$, i.e., close to $H_F$ (Figure 3). Wang et al. [2005] updated this series and found typical fluctuations $\approx 4-5$ times lower (Figure 2). A little later an intermediate (but still sunspot based) estimate yielded a variation of 0.1% since the Maunder minimum, again with $\xi(2)/2 \approx 0.4$ [Krivova et al., 2007] and [12]. The situation changed dramatically with the $\approx 9$ kyr long reconstructions of Steinilhber et al. [2009] and Shapiro et al. [2011]. Both used ice core $^{10}$Be concentrations to estimate the flux of cosmic rays, itself a proxy for the solar magnetic field and hence of solar activity. Although both were calibrated using the satellite observations, their assumptions were quite different, notably about a hypothetical “quiescent” solar state. The $S_2(\Delta t)^{1/2}$ for these reconstructions are remarkable for two reasons. First, they differ from each other by a large factor ($\approx 8-9$, see Figure 2); second, their slopes are the opposite to the sunspot based estimates: rather than $\xi(2)/2 \approx H \approx 0.4$, they have $\xi(2)/2 \approx 0.4 + 0.3!$ While the large factor between them attracted attention, the change in the sign of $H$ was not noticed even though it is probably more important as it would imply an amplification mechanism that increase quite strongly with scale.

Another important driver is explosive volcanism. Volcanoes mainly influence the climate through the emission of sulphates that reflect incoming solar radiation; stratospheric sulphates can persist for months or years after an eruption. The two main volcanic reconstructions [Crowley, 2000; Gao et al., 2008] are based on ice core particulate concentrations. First, sulphate concentrations are estimated and then with the help of models the corresponding global radiative forcings are determined; for $S_2(\Delta t)^{1/2}$, see Figure 2. It is remarkably similar to that of the $^{10}$Be solar variabilities with $\xi(2)/2 \approx -0.3$, it nearly coincides with $S_2(\Delta t)^{1/2}$ from the Shapiro et al. [2011] solar reconstruction. The slightly longer (1500 yrs) Gao et al. [2008] series was converted into equivalent radiative forcings by scaling the mean to the Crowley [2000] series, the $S_2(\Delta t)^{1/2}$ results for the two series are very similar (Figure 2). Although very strong at small $\Delta t$, the volcanic forcings decrease rapidly at longer intervals so that any mechanism responsible for temperature fluctuations must on the contrary involve an amplification that strongly increases with scale.

3. Stochastic and Scaling Climate Sensitivities

[14] We would like to be able to compare the $T$ and $R_F$ fluctuations (Figures 1 and 2) but strictly speaking, the deterministic definition (equation (1)) doesn’t allow it. To interpret our forcing and temperature statistics it is therefore convenient to introduce a stochastic definition of climate sensitivity:

$$\Delta T \overset{d}{=} \lambda R_F$$

where, “$\overset{d}{=}”$ means equality in the sense of random variables the random variables $a, b$ satisfy $a \overset{d}{=} b$ if and only if $Pr(a > s) = Pr(b > s)$ for all $s$, “Pr” means “probability”). Notice that while both deterministic and stochastic definitions
(equations (1) and (3)) predict that the statistical moments
are related by the equation $\langle \Delta T^3 \rangle \propto N^3(\Delta R_F)^3$, the stoch-
astic definition doesn’t even require that $R_F$ and $T$ be
independent. A conventional interpretation is to regard the stoch-
astic $\lambda$ with equality in case of full (and causal)
correlation. The advantage of adopting equation (3) is that
by fixing $\lambda$, we may convert Figure 2 into equivalent tem-
perature fluctuations; Figure 3 shows the resulting super-
positions using $\lambda = 4.5$ K/(W m$^{-2}$) throughout. To put this
value in perspective, we can compare it to $\lambda_0 \approx 0.3$ K/
(W m$^{-2}$), the sensitivity of the simplest energy balance
model involving a homogenous atmosphere and radiative
equilibria. We see that a (large) “feedback” factor $f = \lambda / \lambda_0 =
4.5/0.3 \approx 15$ is necessary to justify the overlaps shown in the
figure.

From equation (3) - and for simplicity only consider-
ing the mean ($q = 1$) behaviour - we see that if $\langle \Delta T(\Delta t) \rangle \propto
\Delta t^\alpha$ and $\langle \Delta R_F(\Delta t) \rangle \propto \Delta t^\beta$, then $H_T = H_T - H_{RF}$. If we
take $H_{RF} \approx -0.3$ (volcanic and 10Be solar estimates), $H_{RF}$
$\approx 0.4$ (sunset based solar) and $H_T \approx 0.4$, then we find $H_T \approx
0.7$ and $\approx 0$ respectively. From Figure 2 we see that the vol-
canic and Shapiro et al. [2011] solar forcings require a $f$
time scale to reach roughly $0.3$ at 30 year scales, rising to roughly $0.7$
at 10 kyr. If we consider instead the scale independent
amplification factors ($H_T \approx 0$), i.e., the Krivova and Wang
reconstructions, we find the (scale independent) factors
$f \approx 15$, 30 respectively. However, for this to apply to multi-
timillennial scales, solar variability must continue to grow
beating $\approx 1$ W m$^{-2}$ at 10 kyr scales.

4. Conclusions

As decreasing over several decades of scale, to a
minimum of $\approx 0.1$ K at around 10–100 yrs, temperature
fluctuations begin to increase, ultimately reaching $3$ to
$5$ K at glacial-interglacial scales. In order to understand the
origin of this multidecadal, multicentennial and multi-
millennial variability, we empirically estimated the climate
sensitivities of solar and volcanic forcings using several
reconstructions. To make this practical, we introduced a
stochastic definition of the sensitivity which could be
regarded as an upper bound on the usual (deterministic)
sensitivity with the two being equal in the case of full (and
causal) correlation between the temperature and driver.
Although the RMS temperature fluctuations increased with
scale, the RMS volcanic and 10Be based solar reconstruc-
tions all decreased with scale, in roughly a power law
manner. If any of these reconstructions represented dom-
inant forcings, the corresponding feedbacks would have to
increase strongly with scale (with exponent $H_T \approx 0.7$), and
this is not trivially compatible with existing GCM’s. Only
the sunspot based solar reconstructions were consistent with
scale independent sensitivities ($H_T \approx 0$), are of the
order $4.5$ K/(W m$^{-2}$) (i.e., implying large feedbacks) and
would require quite strong solar forcings of $\approx 1$ W m$^{-2}$ at
scales of 10 kyr.

A recent analysis of $S_3(\Delta t)^{1/2}$ for forced GCM out-
puts over the past millennium S. Lovejoy et al. (Do GCM’s
predict the climate… Or low frequency weather?, submitted
to Nature Climate Change, 2012) showed that they strongly
underestimate the low frequency variability - even when for
example strong solar forcings were used. Our findings here
of the occasionally surprising scale-by-scale forcing vari-
ability helps explain why they were too weak. It seems
likely that GCM’s are a missing an important mechanism of
internal variability. A possible candidate is land-ice whose
fluctuations are plausibly scaling over the appropriate ranges
of space-time scale but which is not yet integrated into
existing GCM’s.

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