Joint horizontal-vertical anisotropic scaling, isobaric and isoheight wind statistics from aircraft data

J. Pinel¹, S. Lovejoy¹, D. Schertzer², A. F. Tuck³

¹Physics McGill University 3600, University st., Montreal, Que., Canada

²Université Paris-Est, Ecole des Ponts ParisTech, LEESU, Marne-la-Vallée, France

³Physics Department, Imperial College London, London, UK
Abstract

Aircraft measurements of the horizontal wind have consistently found transitions from roughly $k^{-5/3}$ to $k^{-2.4}$ spectra at scales $\Delta x_c$ ranging from about 100 – 500 km. Since drop sondes find $k^{-2.4}$ spectra in the vertical, the simplest explanation is that the aircraft follow gently sloping trajectories (such as isobars) so that at large scales, they estimate vertical rather than horizontal spectra. In order to directly test this hypothesis, we used over 14500 flight segments from GPS and TAMDAR sensor equipped commercial aircraft. We directly estimate the joint horizontal-vertical ($\Delta x, \Delta z$) wind structure function finding - for both longitudinal and transverse components - that the ratio of horizontal to vertical scaling exponents is $H_z \approx 0.57 \pm 0.02$, close to the theoretical prediction of the 23/9D turbulence model which predicts $H_z = 5/9 = 0.555\ldots$. This model also predicts that isobars and isoheight statistics will diverge after $\Delta x_c$; using the observed fractal dimension of the isobars ($\approx 1.79 \pm 0.02$), we find that the isobaric scaling exponents are almost exactly as predicted theoretically and $\Delta x_c \approx 160, 125$ km, (transverse, longitudinal). These results thus give strong direct support to the 23/9D scaling stratification model.
The classical laws of turbulence exploit the scale invariance of the dynamical equations to predict the scaling behaviour of the wind and other turbulent fields. For simplicity, they also assume statistical rotational invariance: isotropy. When applying these laws to the strongly stratified atmosphere, one faces a choice: to drop either the scaling or the isotropy symmetry. Starting with the claimed discovery of the meso-scale gap [Van der Hoven, 1957], and the subsequent development of theories of 2D (isotropic) turbulence [Kraichnan, 1967] - and especially Charney’s geostrophic variant [Charney, 1971] - the dominant choice was to drop the scaling symmetry, to assume that the small scale dynamics were 3D isotropic and the large scale 2D isotropic with a scale break somewhere near the atmospheric scale height (≈10 km).

Starting in the early 1980’s the opposite proposal was made [Schertzer and Lovejoy, 1985b, 1987]: to drop isotropy but to maintain wide range horizontal scaling. In this framework, the vertical structure was also expected to be scaling but with different exponents than the horizontal. Since then, evidence in the horizontal and vertical from satellites, lidar, aircraft, radiosondes, drop sondes and reanalyses has accumulated, supporting the anisotropic scaling model (see the review [Lovejoy and Schertzer, 2010] and also [Tuck, 2008]). More recently, an (anisotropic) scaling (rather than a traditional scale) analysis of the governing equations [Schertzer et al., 2012] has allowed the derivation of new fractional vorticity equations with anisotropic scaling solutions.

Until recently, the outstanding piece of evidence supporting the isotropic 2D/3D model and potentially falsifying the anisotropic scaling hypothesis was the observed break in aircraft spectra of the horizontal wind at scales of several hundred kilometers. However, using high quality scientific aircraft data, [Lovejoy et al., 2004, 2009a] argued that the aircraft trajectories - and hence the wind measurements - would be affected by the turbulence and they predicted a transition from $k^{-\beta_h}$ spectra with $\beta_h \approx 5/3$ and $\beta_{\text{small}} \approx \beta_h$ at small scales to $k^{-\beta}$ spectra at large scales where the aircraft essentially...
sensed the vertical rather than horizontal fluctuations; with vertical exponent $\beta_v \sim 2.4$ and $\beta_{\text{large}} \sim \beta_v$. They also showed that essentially all the horizontal wind spectra and structure functions published to date were compatible with this transition – but not with the more drastic transition from $\beta_{\text{small}} = 5/3$ to $\beta_{\text{large}} = 3$ near 10 km predicted by the competing 2D/3D model.

The paper by [Lovejoy et al., 2009a] sparked a debate [Lindborg et al., 2009, 2010; Lovejoy et al., 2009b, 2009c, 2010; Schertzer et al., 2011, 2012; Yano, 2009] and provoked [Frehlich and Sharman, 2010], (hereafter FS) to perform a new analysis using Tropospheric Airborne Meteorological Data Reporting (TAMDAR) commercial aircraft data. The key new element was that the TAMDAR data had GPS altimetry and were thus – for the first time for commercial aircraft – able to adequately distinguish isobaric and isoheight statistics. This is important because most aircraft follow isobars and these are gently sloping. If these slopes are large enough, then the aircraft spectra will show a spurious transition from $\beta_h$ to $\beta_v$ at a scale which depends on the slope and the turbulent fluxes, thus explaining the observations. FS found neither a scale break near 10 km nor a structure function with exponent anywhere near 2 (corresponding to $\beta_{\text{large}} = 3$) – so that presumably there was not a 2D/3D transition. However, they did make the strong claim that the statistics on isobars and isoheights were identical. If their claim was true, then another mechanism to account for the $k^{-5/3}$ to $k^{-2.4}$ transition would be required.

However, distinguishing the statistics on isoheights and isobars requires very high accuracy – both of wind but especially altitude – measurements. These accuracy requirements are too demanding for the older Aircraft Meteorological Data Relay (AMDAR) equipment (also discussed by FS). With the newer GPS equipped TAMDAR data, the requisite accuracy is possible to achieve if two conditions are met. First, we do not use wind differences from two different aircraft since this involves both larger (absolute) errors as well as nontrivial complications due to the very inhomogeneous distribution of TAMDAR flights paths over the US: the errors are unacceptably large. Second, the TAMDAR sampling protocol was ill adapted for our purpose, it was essential to use only the high quality “cruise”
flight segments. Unfortunately, the copiously sampled ascent and descent segments had to be discarded because of their unacceptably low vertical resolutions (see figure S3 from supplementary material).

Using two aircraft differences and these low resolution segments, we could accurately reproduce the FS TAMDAR results (see figures S1 and S2 from supplementary material).

Finally, we could mention that [Lovejoy and Schertzer, 2010] examined hydrostatic models and found that they also gave isobaric exponent $\beta = 2.4$ and [Lovejoy and Schertzer, 2011] confirmed this in reanalyses, although with an extra complication due to a strong horizontal (zonal/meridional) scaling anisotropy (i.e. different exponents in orthogonal horizontal directions); so that these data are not appropriate for distinguishing isoheight and isobaric statistics. With these differences, we therefore redid the FS TAMDAR analyses.

**Generalized Scale Invariance**

Turbulent laws are of the general type: $\Delta v = f|\Delta r|^H$, where $\Delta v$ is a fluctuation in a turbulent field $v$, $f$ is a scale by scale conserved turbulent flux, $|\Delta r|=(\Delta x, \Delta z)$ is a horizontal vertical lag vector over which $\Delta v$ is calculated (for simplicity, we consider only a single horizontal component) and $H$ is the scaling (non conservation, mean fluctuation) exponent. Different vertical ($H_v$) and horizontal ($H_h$) exponents result when different conserved turbulent fluxes dominate the corresponding dynamics:

\begin{align}
\Delta v(\Delta x) &= \varepsilon^{1/3} \Delta x^{1/3} \\
\Delta v(\Delta z) &= \phi^{1/5} \Delta z^{3/5}
\end{align}

where $\varepsilon$ and $\phi$ are the energy and buoyancy variance fluxes (i.e. $f = \varepsilon^{1/3}, \phi^{1/5}$ respectively). The horizontal law is the Kolmogorov, 1941 scaling and the vertical is the Bolgiano-Obukhov, 1959 scaling.

The way to deal with this anisotropy while keeping an overall scaling symmetry is to replace the usual vector norm for the spatial separation by a different measure of scale – the (anisotropic) scale function – a simple example for vertical section is:
\[
[[\Delta r]] = l_s \left\{ \left| \Delta x/l_s \right| + \left| \Delta z/l_s \right|^{1/H_z} \right\} 
\]

where \( H_z = H_v/H_h = 5/9 \) and \( l_s \) is the “sphero-scale”: the scale at which structures are “roundish”. (If needed, the scale function can be generalized for full space-time vector displacements). The anisotropy is reflected by the exponent \( H_z \neq 1 \) that describes the stratification of structures. Since \( H_z < 1 \), at scales much smaller than \( l_s \), structures tend to be vertically aligned whereas at scales much larger than \( l_s \), they become horizontally flatter. With this scale function, we can write:

\[
\Delta v(\Delta x) = \varepsilon^{1/3} [[\Delta r]]^{1/3} \]  (3)

which, for \(|\Delta r| = (\Delta x, 0)|, |\Delta r| = (0, \Delta z)| \) reduces to eq.1. The vertical extent of structure of horizontal size \( L \) is \( L^{H_z} \); their volumes are \( L^{D_{el}} \) with \( D_{el} = 2 + H_z = 23/9 \); this is the 23/9D model [Schertzer and Lovejoy, 1985a,b]).

Data Analysis

TAMDAR equipped aircraft make short range flights at low altitudes mostly below 7 km; their sensors are the most modern in the AMDAR programme [Moninger et al., 2008; Daniels et al., 2004] and were designed to measure atmospheric fields including wind, humidity, pressure and temperature, as well as location, time and altitude from a built-in GPS. The sampling protocol is important to understand: the system either makes measurements due to significant changes in pressure (changes of 10 hPa or 50 hPa, depending on the altitude) or – if cruising at nearly constant pressure – it switches to a time-based protocol, making measurements every 3 or 7 minutes (again, depending on the level). For an aircraft flying at 500 km/h at an altitude of 5.5 km, the former corresponds to ~25 km. We analysed data for the year 2009 over roughly the continental US (20°N to 50°N latitude). In order to have good statistics and to minimize the strong altitude dependence, we confined our analysis to the layer between 5 and 5.5 km altitude using over 14500 aircraft legs. Only the highest quality data (according to automated quality control checks) were kept.
A nonobvious problem arises since the data were passed through a 10 second smoother, so that, measurements at 250 km/h and angle of 15° correspond to a section 180 m thick. Including these low resolution segments led to biases of 7% at 200 km, but this rapidly increased to 67% at 400 km, hence we discarded them (supplementary material figure S3). This bias, their use of multi-aircraft data pairs and the fact that FS took much thicker layers for isobars and isoheights (4 hPa, 200 m) compared to those used here (1.26 hPa, 20 m) led to our qualitatively different conclusions (supplementary figures S1, S2). Similarly to FS, we took only time intervals less than 1 hour to limit the effects of noninstantaneous measurements.

From the near-constant altitude and near-constant pressure levels, we estimated second order structure functions \( D_n = \langle |\Delta v_i(\Delta r)|^2 \rangle = \langle |v_i(r + \Delta r) - v_i(r)|^2 \rangle \) where \( \langle \rangle \) means ensemble average, \( i = N, L \) for transverse, longitudinal components respectively. The accuracies were estimated from the structure function at small enough lags; per component, the absolute calibration error is \( \approx \pm 1.8 \) m/s, the relative calibration error is \( < \pm 1.8 \) m/s and the altitude error is \( \approx \pm 3 \) m (close to the manufacturer's values \( \pm 2-3 \) m/s on wind speed and \( \pm 3 \) m on altitude, [Daniels et al., 2004]). To eliminate the absolute calibration errors and numerous other problems introduced by the highly nonuniform distribution of TAMDAR trajectories, we always computed velocity increments with data coming from the same aircraft.

**Results**

Taking ensemble averages of the square of eq.3, we obtain:

\[
\langle |\Delta v(\Delta r)|^2 \rangle = \langle \epsilon^{2/3} \rangle [\langle [\Delta r]^2 \rangle = \langle [\Delta v_1/l_{x}]^2 \rangle \langle [\Delta v_1/l_{x}]^2 \rangle \rangle \approx (2) \quad (4)
\]

where \( \zeta_2(2) \) is the second order structure function exponent which takes into account the intermittency of \( \epsilon \). Since \( \langle \epsilon^{2/3} \rangle \sim [\langle [\Delta r]^2 \rangle]^{-K_{\epsilon}(2/3)} \), we have \( \zeta(2) = 2H_{\epsilon} - K_{\epsilon}(2/3) \) where \( K_{\epsilon}(2/3) \) is a small intermittency exponent (\( \sim -0.07 \), see [Lovejoy et al., 2010]). From its definition and the assumption of statistical
translational invariance, $\left\langle |\Delta v(\Delta r)|^2 \right\rangle = \left\langle |\Delta v(-\Delta r)|^2 \right\rangle$; we also assumed left-right symmetry so that the four quadrants of $\left\langle |\Delta v(\Delta x, \Delta z)|^2 \right\rangle$ are symmetric. In order to test the theory, we estimated the parameters in eq. 4 by regression. First, $\zeta_h(2)$ was estimated from linear regression using 1D structure functions (figure 3a), yielding $\zeta_{h,N}(2) = 0.81 \pm 0.02$ and $\zeta_{h,L}(2) = 0.76 \pm 0.03$ which are close to the Kolmogorov value corrected for intermittency $2H_h + 0.07 \approx 0.74$. Only vector lags with at least 500 independent aircraft $\Delta v^2$ estimates were used, the average number over the regression range $16 \text{ km} < \Delta x < 400 \text{ km}$ – see fig. 3 – was 24800. Since presumably $\zeta_{h,N}(2) = \zeta_{h,L}(2)$, we took the value $\zeta_h(2) = 0.8$. Then, from multivariate regression on the joint lags (cross-section, fig. 1), we obtained $H_{z,N} \sim H_{z,L} \sim 0.57 \pm 0.02$, $l_{s,N} \sim l_{s,L} \sim 1.0 \pm 0.1$ mm and $\left\langle |\Delta v(l_s)|^2 \right\rangle^{1/2}_N \sim 3.2 \pm 0.2 \text{ mm/s}$, $\left\langle |\Delta v(l_s)|^2 \right\rangle^{1/2}_L \sim 2.0 \pm 0.2 \text{ mm/s}$. While $H_z$ is close to the theoretical value $H_z = H_h/H_v = (1/3)/(3/5) = 0.56$, the sphaero-scale is a bit smaller than the one estimated ($l_s \sim 4-80 \text{ cm}$) by [Lilley et al., 2004; Lovejoy et al., 2009a; Lovejoy and Schertzer, 2010]. From $H_z$, we can estimate the vertical scaling exponent $\zeta_{v,N}(2) = \zeta_{h,N}(2)/H_{z,N} = 1.42 \pm 0.06$ and $\zeta_{v,L}(2) = \zeta_{h,L}(2)/H_{z,L} = 1.33 \pm 0.07$ values consistent with direct estimates of vertical exponents ($\zeta_v(2) \sim 1.35$ at 6 km) from drop sondes by [Lovejoy et al., 2007]. Interestingly, while both the horizontal and vertical $\zeta(2)$ are little larger than the theoretical values (ignoring intermittency, $2/3$, $6/5$ respectively) yet, as expected, their ratio $H_z$ is almost the same $0.8/1.4 = 0.57$. These exponents are far from the theoretical value of 2D isotropic turbulence $\zeta(2) = 2$. The overall fits (for $|\Delta z| < 40 \text{ km}$ and $|\Delta x| < 275 \text{ km}$) are shown in figure 1, they are very good with mean deviations $\pm 6\%$ and $\pm 4\%$ (transverse, longitudinal respectively). Although the vertical range of scales is short, to our knowledge, figure 1 constitutes the first direct estimate of the joint horizontal-vertical structure function; and it gives strong support to the hypothesis of horizontal-vertical anisotropic scaling.

Fractal aircraft trajectories
In order to compare statistics at constant pressures and constant altitudes, we need to take into account the fractality of the aircraft trajectories. This fractality arises because aircraft at cruising altitudes fly on roughly isobaric levels and these are fractal \cite{Lovejoy et al., 2004}, (although, due to inertia, at scales < 3 km, the trajectories become smooth). This implies:

\[ \langle |\Delta z(\Delta x)| \rangle \sim \left( \frac{\Delta x}{L_f} \right)^{H_{tr}} \langle |\Delta z(L_f)| \rangle \]  (5)

where \( \langle |\Delta z(\Delta x)| \rangle \) is the average vertical displacement of an aircraft over a horizontal lag \( \Delta x \), \( L_f \sim 180 \) km is the average length of our TAMDAR flights, (chosen as a convenient reference scale) and \( H_{tr} \) =\( D_{tr} - 1 \) where \( D_{tr} \) is the fractal dimension of the “trajectory” (more precisely, it corresponds to the fractality our isobaric sampling). From figure 2, we find \( H_{tr} = 0.79 \pm 0.02 \) and \( \langle |\Delta z(L_f)| \rangle \sim 19 \pm 2 \) m (which represents the average vertical displacement of an aircraft over \( L_f \)).

To investigate the consequences for the velocity fluctuations statistics, we can use eq. 2 and make a rough “mean field” type argument (see \cite{Lovejoy et al., 2009a}), where, in the scale function (eq.2), we replace \( \Delta z \) with \( \langle |\Delta z(\Delta x)| \rangle \) from eq.5:

\[ \langle |\Delta v(\Delta x)|^2 \rangle \sim \left[ \frac{\Delta x}{l_s} + \frac{\Delta x}{\Delta x_0} \right]^{\zeta_{v,2}} \Delta x_0 = L_f \left( \frac{l_s}{\langle |\Delta z(L_f)| \rangle} \right)^{1/H_{tr}} \]  (6)

Since \( H_{tr} > H_z \), for isobars, there is a critical scale \( \Delta x_c \) for which, (taking \( \zeta_{v,2} = \zeta_{h,2}/H_z \approx 0.8/0.57 \approx 1.4 \)):

\[ \langle |\Delta v(\Delta x)|^2 \rangle \sim |\Delta x|^{\zeta_{v,2}} \quad \Delta x \ll \Delta x_c \] ; \( \Delta x_c \sim (\Delta x_0 l_s)^{1/(H_{tr} - H_z)} \)  (7)

\[ \langle |\Delta v(\Delta x)|^2 \rangle \sim |\Delta x|^{\zeta_{v,2}} \quad \Delta x \gg \Delta x_c \]

The value of \( \Delta x_c \) depends on the turbulent fluxes (through the parameter \( l_s \)) and on \( \Delta x_0 \). Figure 3 (top) compares the horizontal structure functions for near-constant pressure and altitude levels. As expected, the two curves are nearly identical for scales smaller than \( \Delta x_{c,N} \sim 160 \) km, \( \Delta x_{c,L} \sim 125 \) km, and follow a
straight line with slope $\zeta_h(2) \approx 0.8$. As predicted, for scales larger than $\Delta x_c$, the near-isobaric (orange) curve follows a new line with slope $H_{tr}\zeta_v(2) \approx 1.1$. At the extreme large scale limit of our data (~300 km), there is a small deviation in the scaling of the longitudinal component. We checked that at this scale, there was a 25% difference in the contribution to $\langle |\Delta v_L(\Delta x)|^2 \rangle$ for positive and negative $\Delta v_L$; since the aircraft mostly made round trips, this must be a consequence of the pilot modifying the trajectories depending on the weather – particularly affecting longitudinal components – hence introducing correlations between the aircraft and wind. By taking the ratio of the isobaric and isoaltitude $\langle |\Delta h|^2 \rangle$ we largely eliminate this effect (figure 3b): as predicted, the isoheight to isobar ratio continues to grow with scale.

This $2+H_z=2.57$ dimensional turbulence has transverse to longitudinal ratio $D_{NN}/D_{LL} \approx 1.78 \pm 0.08$, somewhat higher than the theoretical 3D, 2D isotropic turbulence values ($D_{NN}/D_{LL} \approx 4/3$, 5/3, respectively [Monin and Yaglom, 1975; Ogura, 1952; Lindborg, 1999]).

In order to further test the 23/9D theory, we show the results for data pairs constrained to have slopes $> 3 \times 10^{-4}$ (roughly the mean isobaric slope at 400 km resolution), these trajectories are not fractal ($H_{tr} \approx 1$, top, figure 2) so that we expect exponents $H_{tr}\zeta_v(2) \approx 1.4$. This is confirmed by figure 3a (top) at scales $> 40$ km. For these conditional isobaric curves, figure 3a indicates $\Delta x_c \approx 36$ km, a value close to $\Delta x_c \approx 40$ km estimated using eq.7 and parameters estimated on figures 1 and 2. The bottom of figure 3 shows the difference between isobaric (with and without the condition on the aircraft slopes) and near-constant altitude cases. For increasing horizontal lags, the difference between the isobaric and near-constant altitudes curves increase, showing the relevance of the 23/9D model and the effect of fractal trajectories as described by (eq.5). Interestingly, the previous studies cited (from flights near the top of the troposphere, including fig. 1, 2 of FS) find $\zeta(2) \approx 1.4$ so that presumably for these, $H_{tr} \approx 1$.

Conclusions:
The horizontal wind field is anomalous in that it has a break in the scaling at scales typically in the range 100 – 1000 km with small scale spectra roughly $k^{-5/3}$ transitioning at lower wavenumbers to $k^{-2.4}$. Both the transition scale and exponent are quite different from those predicted by theories of isotropic 3D and isotropic 2D turbulence ($\approx 10$ km and $k^{-3}$). A simple explanation is that the aircraft trajectories are gently sloping (e.g. they are isobaric) so that at a critical scale, the vertical fluctuations are dominant implying $k^{-2.4}$ for the sloping spectra (as in the vertical). In order to test this directly, high accuracy altitude and wind measurements are required; when carefully used the TAMDAR commercial aircraft sensors are adequate. However, due to degraded vertical resolution on ascent and descending flight segments, only cruise altitude data should be used and stringent pressure and altitude bounds are needed to define the isoheights and isobars ($\pm 10$ m, $\pm 0.63$ hPa).

Using data from over 14500 flights, for the first time we were able to estimate the joint horizontal-vertical structure functions providing strong support to the 23/9 D anisotropic scaling theory (fig. 1), and estimating the key stratification exponent as $H_z=0.57\pm 0.02$, quite close to the theoretical value 5/9. Using this, and the observed fractal dimension of the isobars ($D_{fr}=1.79\pm 0.02$), we were able to theoretically calculate the isoheight, isobaric and constant slope structure function exponents (0.8, 1.1, 1.4 respectively) as well as the critical isoheight/isobar transition distance ($\approx 160$ km, 125 km, transverse, longitudinal). The results of this study give the strongest and most direct support to date for the 23/9D anisotropic scaling model.
References


Frehlich, R. G., R. D. Sharman, (2010): Equivalence of velocity statistics at constant pressure or constant altitude, GEOPHYSICAL RESEARCH LETTERS, VOL. 37, L08801, 5 PP.


measurements in anisotropic scaling turbulence” by S. Lovejoy et al. (2009)” Atmos. Chem. Phys.

Disc., 9, C9797-C9798.


anisotropic scaling turbulence, Atmos. Chem. Phys. 9, 1-19.

measurements in anisotropic scaling turbulence” by S. Lovejoy et al., Atmos. Chem. Phys. Disc., 9,
S2592- S2599.

aircraft measurements in anisotropic scaling turbulence" by Lovejoy et al. (2009)” by E. Lindborg et al,


Van der Hoven, I. (1957): Power spectrum of horizontal wind speed in the frequency range from 0.007 to 900 cycles per hour, J. Meteorol., 14, 160–164.

Figures captions

Figure 1: Contour plot of $\left( |\Delta v(\Delta x, \Delta z)| \right)^2$, in black: horizontal wind measured by TAMDAR. In purple, a fit with the help of the scale function (eq.2). Parameters are: $l_{s,N} \sim l_{s,L} \sim 1.0 \pm 0.1$ mm, $\zeta(2)=0.8$ (from 1D structure functions fits) and $H_{z,N} \sim H_{z,L} \sim 0.57 \pm 0.02$ and $\langle \varepsilon_L^{2/3} \rangle^{3/2} \sim (4.0 \pm 0.8) \times 10^{-6}$ m$^2$/s$^3$. The average percentage of error between fitted and empirical curves, according to $\left( \frac{\langle |\Delta v|_{fit} \rangle - \langle |\Delta v|_{emp} \rangle}{\langle |\Delta v|_{fit} \rangle} \right)$ is $\sim 6\%$ (4\%) for transverse (longitudinal) component.

Figure 2: Mean vertical displacement as a function of horizontal separation. In orange: structure function calculated over near-constant pressure levels ($\Delta p<1.26$ hPa). In purple: structure function calculated over near-constant pressure levels with an additional constraint on the slope ($\Delta z/\Delta x > 3 \times 10^{-4}$). Reference lines have slopes 0.8 and 1.0. The near-constant altitude (orange) curve shows a break in scaling symmetry for $\Delta x<16$ km due to poor statistics.

Figure 3: a) $\left( |\Delta v(\Delta x, \Delta z)| \right)^2$ for the transverse (upper curves) and longitudinal (lower curves) components of the wind measured by TAMDAR. The curves for transverse components were displaced in the vertical by 0.5 for clarity. In green: $\left( |\Delta v(\Delta x, \Delta z)| \right)^2$ calculated over near-constant altitude levels ($\Delta z<20$ m). In orange: $\left( |\Delta v(\Delta x, \Delta z)| \right)^2$ calculated over near-constant pressure levels ($\Delta p<1.26$ hPa). In purple: $\left( |\Delta v(\Delta x, \Delta z)| \right)^2$ calculated over near-constant pressure levels with an additional constraint on the slope ($\Delta z/\Delta x > 3 \times 10^{-4}$). Reference lines have slopes 0.8, 1.1 and 1.4. b) in orange (red): difference
between log of \( \left\langle |\Delta v(\Delta x, \Delta z)|^2 \right\rangle \) calculated on near-constant pressure and near-constant altitude levels for the transverse (longitudinal) component. In Purple (blue): difference between log of \( \left\langle |\Delta v(\Delta x, \Delta z)|^2 \right\rangle \) calculated on near-constant pressure with the additional constraint on the slope \( (\Delta z/\Delta x > 3 \times 10^{-4}) \) and near-constant altitude levels for the transverse (longitudinal) component. Reference lines have slopes 0.3 and 0.6.
Introduction:

This supplementary material contains complementary figures that could not be included in the final version of the paper.

Figure captions:

Figure S1: Comparison of our analysis with the results obtained by Frehlich and Sharman. We show second order structure function for the transverse component of the wind measured by TAMDAR. Unlike the figures in the paper, here we include the low resolution ascending and descending flights segments; we also compare structure functions from two aircraft (top pair), and single aircraft (bottom pair). This analysis was made on the same latitude band as Frehlich and Sharman (40N-50N) and between roughly the same altitude levels (5km-6km), using the same criteria for near-constant pressure (Δp<4 hPa) and altitude (Δz<200 m) levels. In Blue: data from the same aircraft at near-constant altitude levels. In brown: data coming from the same aircraft at near-constant pressure levels. In green: data coming from different aircraft at near-constant altitude levels. In red: data coming from different aircraft at near-constant pressure levels. The large black dots are reproduced from the Frehlich and Sharman TAMDR analysis.

Figure S2: Same as figure S1, but for longitudinal component.

Figure S3: Difference between log of second order structure functions for cruising parts only and for complete flights, including ascending and descending parts (after take off and before landing). In blue (red): transverse (longitudinal) component. This is for complete year 2009 for latitudes 20N-50N, between 5-5.5 km of altitudes.
$\log_{10} \langle |\Delta v|^2 \rangle \text{(m/s)}^2$

$\log_{10} \Delta x \text{(km)}$