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RESEARCH ARTICLE

Impacts of small scale rainfall variability in urban areas: a case study with 1D and 1D/2D hydrological models in a multifractal framework

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In this paper the sensitivity to small scale unmeasured rainfall variability (i.e. at scales smaller than 1 km by 1 km by 5 min in time, which are usually available with C-band radars) of a 1D/2D model with a 10 m resolution and a semi-distributed 1D model of the same 1.47 km\(^2\) urban area is analyzed. The 1D/2D model is the open source numerical platform Multi-Hydro, which couples (open source) distributed models of involved hydrological/hydraulic processes. The methodology implemented to evaluate the uncertainties consists of generating an ensemble of realistic rainfall fields downscaled to a resolution of 12.3 m in space and 18.75 s in time with the help of a stochastic universal multifractal model. The corresponding ensemble of hydrographs is then simulated. It appears that the uncertainty is significant and that Multi-Hydro unveils much more uncertainty than the simpler 1D model. This points out a need to develop high resolution distributed modelling in urban areas.

Keywords: rainfall variability; 1D/2D modelling; multifractals; space-time downscaling

1. Introduction

Rainfall variability has a significant impact on river discharges (see Singh, 1997 for a review). This impact is enhanced in urban areas where the response times of catchments are shorter and the coefficients of imperviousness are larger meaning that a greater fraction of rainfall is immediately active (Aronica & Cannarozzo, 2000; Segond, Wheater, & Onof, 2007). The under-representation of rainfall variability in input data of models affects the confidence one should have in its predictions. A better understanding of rainfall variability in urban areas and its impact on simulated flow is needed both theoretically and operationally. Indeed real time control (RTC, see Schütze, Campisano, Colas, Schilling, & Vanrolleghem, 2004 for a review of its rapid development over the last decades) of sewer networks, which aims at reducing urban flooding and pollution, relies on the use of such models.

In recent papers Gires et al. (2012a, 2012b) quantified the impact of small scale unmeasured rainfall variability (i.e. at scales smaller than the C-band radar resolution of 1 km \(\times\) 1 km \(\times\) 5 min, which is usually provided by national meteorological services of Western European countries) on urban discharges simulated with the help of semi-distributed urban hydrological / hydraulic 1D models. Two urban areas were studied: a 3400 ha one located near Paris and a 900 ha one located in the north of London. The methodology implemented relied on the generation and analysis of realistic ensembles: (i) generation of an ensemble of realistic rainfall fields through a stochastic multifractal downscaling of the radar data, (ii) simulation of the corresponding ensemble of hydrographs with a semi-distributed 1D model, (iii) quantification of the variability among these ensembles. A limitation of these works was that the size of the sub-catchments (roughly 17 ha on average), which are considered as homogenous objects, did not enable a full grasp on the actual rainfall spatial variability. In this paper we implement the same methodology on a portion of size 144 ha of the previous Paris area case study (see Figure 1).

Two types of models are used: the same semi-distributed operational one and a 1D/2D fully distributed one called Multi-Hydro. It is a numerical platform currently being developed at Ecole des Ponts ParisTech and validated in the framework of FP 7 SMARTeST European Project (v1, El Tabach, Tchiguirinskaia, Mahmood, & Schertzer, 2009; v2, Giangola-Murzyn, Gires, Hoang, Tchiguirinskaia, & Schertzer, 2012). The aim of the paper is mainly to develop a methodology to take into account small scale unmeasured rainfall variability, and test how two different models quantify the associated uncertainty on three rainfall events.

The rainfall event and data are described in Section 2.1. Section 2.2 briefly presents Multi-Hydro. The 144 ha studied urban area denoted Kodak catchment and its
representation with the two models is presented in Section 2.3. Section 2.4 describes the implemented methodology. Results are discussed in Section 3.

2. Data and methods

2.1. Rainfall data

The three rainfall events studied in this paper occurred over the Paris area on 9 February 2009, 15 August 2010 and 15 December 2011. In the following they will simply be denoted 2009, 2010 and 2011 event. The rainfall data is the Météo-France radar mosaic whose resolution is 1 km in space and 5 min in time. The rainfall rate \( R \) is basically computed from the radar reflectivity \( Z \) with the help of a standard \( Z = aR^b \) relationship with \( a = 200 \) and \( b = 1.6 \) (\( Z \) in \( \text{mm}^6\text{m}^{-3} \) and \( R \) in \( \text{mm.h}^{-1} \)). Details about the additional corrections which are implemented can be found in Tabary (2007). The studied catchment is located at approximately 45 km of the C-band radar of Trappes (West of Paris) meaning that the rainfall estimates are still reliable (Tabary et al., 2007). The temporal evolution of the average rain rate over the Kodak catchment for the three events are displayed Figure 2. The total rainfall depth for the 2009, 2010 and 2011 event is respectively 8.3 mm, 56.5 mm and 23.8 mm for durations of approximately 6 h, 30 h and 12 h. These events are heavy ones especially the last two, but over a duration of 1 or 4 hour, none has a return period greater than 1 year (data from a rain gauge located in the Paris area that was available to the authors was used to confirm that).

2.2. Short presentation of Multi-Hydro

There is a growing interest for taking into account more precisely the interactions between surface and sewer flows in the field of urban hydrology (Hsu, Chen and Chang, 2000; Rodriguez, Andrieu, & Morena, 2008; Leandro, Chen, Djordjević, & Savić, 2009; Maksimović et al., 2009;...
In that context, Multi-hydro is a numerical platform currently under development that makes interact several open source software packages each of them representing a portion of the water cycle in urban environment. The second version that is used in this paper consists in an interactive coupling between a 2D model representing surface runoff and infiltration (TREX, Two dimensional Runoff, Erosion and eXport model, Velleux, England, & Julien, 2011) and a 1D model of sewer networks (SWMM, Storm Water Management Model, Rossman, 2007). Only the hydraulic part of SWMM is used to model water flow in pipes, and not the hydrologic one. The main input data is a precise description of the sewer network, the topography, and the land use distribution. In this case study six different classes of land use are used (wood, grass, water, roads, building and gullies), each being fully characterized by its hydraulic conductivity (m/s), capillary suction (m), moisture deficit (no unit, ranging from 0 to 1), Manning coefficient (s.m$^{-1/3}$) and depth of interception (mm). With regards to the land use distribution only one class can be affected to each pixel. Therefore, as a pixel usually contains several types of land use, an order of priority must be set to determine the land use of a pixel actually containing several ones (in the vector GIS data). The order set here is gully, road, buildings, water, wood and grass. The influence of this feature will be discussed in the following. Concerning the topography the digital terrain model used was provided by the Institut National de l’Information Géographique et Forestière (http://professionnels.ign.fr/), and does not take into account anthropogenic elevation modification (buildings are removed from the raw data coming from field measurements). The elevation used in Multi-Hydro is an interpolation of this data whose initial spatial resolution is 25 m with a vertical resolution of 1 m. As a consequence, the raw elevation of the road pixels is decreased by 15 cm and the building pixels one is increased by 5 m to prevent water from running through these pixels. The rainfall collected by the building pixels is directly routed to the nearest gully. The interactions between the stormwater sewer system and surface flow are handled through the gullies where water can circulate in both ways, i.e. from surface to sewer in standard situation, and the other way in case of sewer overload. More details about Multi-Hydro can be found in Giangola-Murzyn et al. (2012). The data is formatted for Multi-Hydro from commonly available GIS data with the help of an in-house developed tool called MH AssimTool (Richard, Tchiguirinskaia, & Schertzer, 2012). This enables to (rather) easily implement the model on a new catchment.

2.3. **Studied catchments and their representation with the help of two models**

The catchment (denoted Kodak catchment after) which is mainly studied, i.e. where the impact of small scale rainfall variability is tested, is a roughly 1.4–1.5 km$^2$ urban area located in the city of Sevran (Seine-Saint-Denis county, North-East of Paris). The area is rather flat with an elevation difference of only 11 m between the highest point and the outlet. There is a separate sewer system in the area, and the storm water drainage system regularly overflows, hence a project to build a storm water storage basin to limit it. The basin will also enable to reduce water transfer during heavy rainfall to the downstream area just north of it which suffers frequent pluvial flooding. There is a flow gauge in the conduit above the park which corresponds to a former Kodak factory (see Figure 1). It drains all the East part of the catchment (about 2/3 of the total area). The operators of the gauge told us that the accuracy of the flow measured by this gauge is not very good because it is located just downstream a screen. It was actually initially installed to study the flow features just downstream this screen. There is unfortunately no data available to assess the uncertainty associated with this measurement. Figure 3b displays the land use distribution for pixels of size 10 m × 10 m for which most of the study will be carried out, along with the storm water drainage system (waste water sewers are not modelled in this study).

The Direction Eau et Assainissement de Seine-Saint-Denis (DEA 93, the local authority in charge of urban drainage) calibrated and runs operationally the semi-distributed 1D model Canoe (Allison, INSA Lyon, & SOGREAH Consultants, 2005) on this area. In Canoe the hydrologic response of each sub-catchment is modelled with the help of a lumped model (a linear reservoir) and the flow in the pipes is modelled with the help of a numerical solution of Saint-Venant equations. The studied area (see Figure 3d) is divided into 16 sub-catchments whose size is ranging from 4–14.5 ha. The total area studied with the 1D model is 1.39 km$^2$ (the difference with Multi-Hydro comes from side effects), and the average coefficient of imperviousness is equal to 53%.

Another catchment called “Loup catchment” is studied in this paper to validate the Multi-Hydro model. It is a 0.5 km$^2$ area also located in Seine-Saint-Denis County few kilometres North-East of the Kodak catchment. It is mainly an industrial area with a small portion of housing estate. The area is rather flat with an elevation difference of only 12.5 m between the highest point and the outlet, and drained by a separate sewer network. We only model the storm water drainage network. Figure 4 displays its representation in Multi-Hydro with pixels of size 10 m × 10 m, and the storm water sewer network. The outlet of the catchment is the roundabout visible in the South-West portion of the catchment. It is actually a storm water storage basin managed by the DEA 93, which receives water only from this catchment. At the beginning of an event, the outlet gate of the basin is closed, which means that the water volume contained in the basin corresponds to the one generated by the Loup catchment. There is a water level gauge with a precision of roughly 1 cm monitoring in real time the basin. This height coupled with a precise geometrical description...
of the basin enables to plot the temporal evolution of the volume observed in the basin and compare it with the cumulated flow simulated with the help of Multi-Hydro at the outlet of the Loup catchment.

2.4. Methodology

In order to quantify the uncertainty associated with small scale unmeasured rainfall variability, the following methodology is implemented for each event: (i) An ensemble of 100 realistic downscaled rainfall fields with a resolution of 12.3 m in space and 18.75 s in time is generated. For the semi-distributed model, given the size of the homogeneous sub-catchments, the rainfall was only downscaled only to 111 m in space and 1.25 min in time. (ii) The corresponding ensemble of hydrographs is then simulated for each model. (iii) The variability among the hydrographs is characterized with the help of the envelop curves $Q_{0.1}$, $Q_{0.25}$, $Q_{0.75}$ and $Q_{0.9}$, which are respectively made of the 10, 25, 75 and 90% quantiles (in m$^3$/s) estimated for each time step. Finally we compute for the peak flow a pseudo coefficient of variation defined as:

$$CV' = \frac{Q_{0.9}(t_{PF,\text{radar}}) - Q_{0.1}(t_{PF,\text{radar}})}{2PF_{\text{radar}}}$$ (1)

where $PF_{\text{radar}}$ is the peak flow simulated with the raw radar data and $t_{PF,\text{radar}}$ is its time of occurrence. In this paper, for flow $CV'$ is discussed only for $t_{PF,\text{radar}}$, and it corresponds to the time step for which it the greatest. This is a quantitative indicator of the uncertainty associated with small scale rainfall variability for the peak flow which is of prime importance for urban hydrologists.

The rainfall input downscaling technique relies on the framework of Universal Multifractals (Schertzer & Lovejoy, 1987), which has been extensively used (Schertzer & Lovejoy, 2011 for a recent review; de Lima & de Lima, 2009, and Verrier, de Montera, Barthes, & Mallet, 2010, for applications in hydrology) to analyse and
simulate geophysical fields extremely variable over wide ranges of scales. In this framework it is assumed that rainfall is generated through a space-time multiplicative cascade process characterized with the help of only two parameters; \(C_1\) the mean intermittency (which measures the average sparseness of the field) and \(\alpha\) the multifractality index (which measures the variability of the intermittency when considering intensities slightly different from the average field). The downscaling implemented in this paper simply consists in stochastically continuing the cascade process whose features are assessed over the available range of scales. No data on these events was available to confirm the validity of the multifractal framework down to scale of 12.3 m and 18.75 s. However Mandapaka, Lewandowski, Eichinger, and Krajewski (2009) showed with the help of Lidar data that rainfall exhibited a scaling behaviour down to 1 m in space and 1 s in time. Discrete cascades are used. The UM parameters used here are \(\alpha = 1.8\) and \(C_1 = 0.1\) which corresponds to the ones usually found focusing the analysis on the rainy portion of the rainfall field (de Montera, Barthes, Mallet, & Gole, 2009; Mandapaka et al., 2009; Verrier et al., 2010; Gires, Tchiguirinskaia, Schertzer, & Lovejoy, 2013). More details on the downscaling process can be found in Gires et al. (2012b). A validation of the downscaling model with the help of two dense networks of 16 disdrometers or rain gauges deployed over a 1 km\(^2\) area in respectively Switzerland and United Kingdom is suggested in Gires et al. (2014). More details on the simulation of Universal Multifractal fields can be found in Pecknold, Lovejoy, Schertzer, Hooge, and Malouin (1993) and Lovejoy and Schertzer (2010).

3. Results and discussion

3.1. Models resolution and validation

Before discussing the issue of the validation of the models, it is required to address the question of the resolution of Multi-Hydro, i.e. the size of its pixels. Indeed as mentioned before, Multi-Hydro is developed so that a single land use class is affected to each pixel, and therefore an order of priority is set to determine the class of a pixel. An illustration of this feature is given in Figure 3 which displays the land use distribution obtained with pixels of various sizes. Striking differences are visible. For example the gardens attending the houses are almost not visible with pixels of size 20 m \(\times\) 20 m whereas they are with pixels of size 1 m \(\times\) 1 m. These differences result in hydrological consequences. An illustration is the percentage of impervious area (gully, road and building pixels), which reflects the portion of storm water rapidly active. It is equal to 87, 83, 77, 63, 53, 47, and 40\% respectively for pixels of size 20, 15, 10, 5, 3, 2, and 1 m. The size of the modelled area ranges from 1.49 km\(^2\) with a 20 m pixels to 1.42 km\(^2\) with 1 m pixels. It is interesting to note that such behaviour is rather standard of a fractal set. Such set is characterized by a fractal dimension \(D_F\) defined with the help of the following equation:

\[
N_x = \lambda^{D_F}
\]

where \(N_x\) is the number of boxes of size \(l\) needed to completely cover it and \(\lambda\) is the resolution (\(\lambda = L/l\), with \(L\) the outer scale of the set). Here \(N_x\) was computed from the impervious pixels of a 1024 m \(\times\) 1024 m area of the 1 m grid (Figure 3a). The straight line (\(R^2\) greater than 0.99) reflects that it is a fractal set, and the slope equal to 1.85 corresponds to the fractal dimension (Figure 5). The fact that the geometrical set of impervious areas exhibits a fractal behaviour suggests that such tool should be used more frequently in order to first characterize urban environment and then model it. This is nevertheless not the scope of this paper, which focuses on the rainfall input, to investigate more in-depth this issue.

This feature of one single land use class per pixel is a limit of Multi-Hydro, but also a strength since this simple rule enables to develop an automatic process to generate input data from available GIS data which make the model easily transportable. The selected resolution of Multi-Hydro results from a trade off between the computation time (which increases non-linearly with decreasing pixel size), the quality of the available land use distribution (a non – obvious issue at high resolution in urban areas!) and the desired accuracy according to the application. In this paper Multi-Hydro is used with pixels of size 10 m \(\times\) 10 m and 5 m \(\times\) 5 m for the Kodak catchment, and only 10 m \(\times\) 10 m for the Loup catchment. Multi-Hydro is implemented without any calibration, i.e. standard values for the 5 parameters describing a land use class are used (Giagola-Murzyn et al. 2012). For the Kodak catchment the resolution of Canoe corresponds roughly to pixels of size 300 m (obtained simply by taking the square root of the catchment area divided by the number of sub-

![Figure 5. Estimation of the fractal dimension (Equation (2) in a log-log plot) of the impervious portion of the Kodak catchment.](image-url)
catchments). The average coefficient of imperviousness is equal to 53% which roughly corresponds to the value found with pixels of size 3 m in Multi-Hydro.

Figure 6 displays the temporal evolution of the flow simulated with the different models (Multi-Hydro 10 m and 5 m, and Canoe) and the flow measurements (see Figure 1 for the location of the flow gauge) for the Kodak catchment and the 2009 event (the only one for which flow measurements are available). This rainfall event did not generate any storm water sewer overflow. The curves for the different models exhibit rather comparable patterns. The differences in terms of numerical values are essentially due to the variations of the percentage of impervious area. The time of peak flow is similar for all these curves with less than 5 min shift. Concerning the comparison with the measurements, the Nash-Sutcliffe coefficient is equal to 0.40 for MH 10m, 0.68 for MH 5 m, and 0.78 for Canoe. The three models react too quickly at the beginning of the rainfall which is likely to be due to a misrepresentation of the initial losses. The three models also miss the first measured peak (slightly before 5 h of simulations). There is no clear explanation for this, but it could be due to errors in the rainfall measurement (possible for this event, see comments on next paragraph) or the flow measurement which are known to be not very accurate here. Anyway more events should be tested to properly validate these models.

Multi-Hydro was also tested with pixels of size 10 m \( \times \) 10 m on the Loup catchment (see Abbes, 2013 for more an extensive study). With this resolution, the percentage of impervious areas is of roughly 90%. No additional calibration was done on Multi-Hydro and the same parameter set as for the Kodak catchment was used. Figure 7 displays the volume measured in the storage tank, and the simulated one for the three events with raw radar data and also with the data (considered homogenous over the catchment) from a rain gauge located 1 km away from the catchment. The simulated volume is the cumulative flow at the outlet. For the 2011 event, the agreement is good. The 2010 event lasted 30 h, hence water was released from the storage tank during that time, which is why three portions had to be selected to compare measurements and simulations. During the first portion, Multi-hydro with both rainfall inputs (radar and rain gauge) overestimates observed volume. During the second one, measurements are in between simulations with radar and rain gauge data. For the third one Multi-Hydro tends to slightly overestimate volume. Except for the first portion, which might reflect issues in the handling of initial loss and watering of surfaces, the agreement between simulations and observations is good. For the 2009 event discrepancies between radar and rain gauge measurements are the greatest and the measurements (until water is released from the tank after 8 h) are between the two simulations. This suggests that there might be some issues with regards to the rainfall estimation for the 2009 event which might explain partially the discrepancies of Figure 6.

The aim of the paper is not to reach the perfect model, but only to have enough confidence in the models so that it makes sense to analyse their sensitivity to the rainfall input resolution. The results of the previous paragraphs show that the models are roughly consistent and that it is therefore legitimate to use them for the purpose of the paper.

3.2. Uncertainty associated with small scale rainfall variability for various rainfall events and the Multi-Hydro model

Figure 8 displays the flow simulated with raw radar data (\( Q_{\text{radar}} \)) and the uncertainty intervals (\( Q_{0.01} \), \( Q_{0.05} \), \( Q_{0.075} \) and \( Q_{0.9} \)) for five conduits obtained for the 2009 event with Multi-Hydro 10 m. The rainfall was downscaled from an initial resolution of 1 km in space 5 min in time to respectively 12.3 m and 18.75 s. The analysis was performed with pixels of size 10 m \( \times \) 10 m even though the simulated flow might be less accurate than with smaller pixels like 5 m \( \times \) 5 m because the computation time for each sample is much smaller (roughly 1 h versus 4 h on standard laptop). Before going on, it should be mentioned that the observed differences between the hydrographs are not due to variations in the total rainfall amount, but to variations in the spatio-temporal distribution of rainfall. Indeed for the 2009 event the raw radar total rainfall amount is of 8.2 mm, whereas it is of 8.3 mm on average with a ratio of the difference between the 95% and 5% quantile and twice the radar total volume equal to 3.4% (this figure defined on the same principle as the pseudo coefficient of variation \( CV^* \) quantifies the variability
among the ensemble) for the generated downscaled rainfall fields. For the 2010 event the corresponding values are respectively 56.5 mm, 56.5 mm, and 1.6%. For the 2011 event the corresponding values are respectively 23.8 mm, 23.9 mm and 3.5%. These disparities are much smaller than the ones observed on the simulated discharges even at the outlet (the smallest computed CV’ is equal to 15%). Figure 8 enables to analyse the uncertainty

Figure 7. Comparison of the cumulated flow simulated with Multi-Hydro and observed for the Loup catchment.

Figure 8. Simulated flow with the raw radar data (black), $Q_{0.25}$ and $Q_{0.75}$ (dark colour), $Q_{0.1}$ and $Q_{0.9}$ (light colour) for 5 conduits of the Kodak catchment with the help of the Multi-Hydro 10 m model for the 2009 rainfall event.
according to the position (i.e. upstream or downstream) of the conduit in the sewer network. As expected the uncertainty increases with upstream conduits. For the February event the computed $CV'$ ranges from 15% for the outlet to 38% for the most upstream selected conduit. These values are rather elevated, and suggest that a better rainfall input would result in a significant decrease on the uncertainty of the simulated flow. It means that a better rainfall input would help local authorities to better cope with real time management of storm water sewer flooding. Moreover these levels of uncertainties are observed for a moderate rainfall event, which was not necessarily expected. It suggests that higher resolution rainfall would also be needed to improve real time management of water quality.

With regards to the other events, Figure 9 displays the flow simulated with raw radar data ($Q_{\text{radar}}$) and the uncertainty intervals ($Q_{0.1}, Q_{0.25}, Q_{0.75}$ and $Q_{0.9}$) at the outlet for the three selected events. For the 2010 event $CV'$ ranges from 21% at the outlet to 61% for the most upstream selected conduit (same as in Figure 8). The values are respectively 18% and 43% for the 2011 event. These results are qualitatively similar to the ones obtained for the 2009 event which confirms the conclusions of the previous paragraph. It even appears that the uncertainty associated with small scale rainfall variability tends to be greater for heavier events.

3.3. Comparison of uncertainty computed by the two models

The flow and its uncertainty simulated with the help of the 1D semi-distributed model at the outlet of the Kodak catchment for the 2009 event is visible Figure 9b. We remind that for the 1D semi-distributed model the rainfall was only downscaled to a resolution of 111 m in space and 1.25 min in time and not to respectively 12.3 m and 18.75 s as for Multi-Hydro, because the sizes of the sub-catchments (ranging from 4 to 14.5 ha) are already much greater than the size of the pixels of the downscaled rainfall field (1.2 ha). It appears that the uncertainty intervals are much larger with Multi-Hydro than for the 1D model, and this during the whole event and not only the peak flow. This is confirmed by $CV'$ which is equal to 15% for the Multi-Hydro 10 m and to 8% for the semi-distributed model. It means that such 1D model is not able to fully take into account the small scale rainfall variability which has been shown to have a significant impact on the simulated flow. It would be interesting to carry out further investigations on 1D models by testing this methodology for configurations with sub-catchments much smaller (1 ha or less) than the ones of this paper. To actually benefit from the higher resolution rainfall data which is becoming increasingly available in urban areas, there is a need to develop the use of a fully distributed model.

Figure 9. Simulated flow with the raw radar data (black), $Q_{0.25}$ and $Q_{0.75}$ (dark colour), $Q_{0.1}$ and $Q_{0.9}$ (light colour) for the outlet of the Kodak catchment. (a) Multi-Hydro 10 m, 2009 event; (b) 1D model, 2009 event; (c) Multi-Hydro 10 m, 2010 event; (d) Multi-Hydro 10 m, 2011 event.
Table 1. Values of computed \( CV' \) for three links (the links are the two extreme ones and the middle one that are selected for Figure 8) for various UM parameters set.

<table>
<thead>
<tr>
<th></th>
<th>( \alpha = 1.8; C_1 = 0.1 ) ( (\gamma_s = 0.50) )</th>
<th>( \alpha = 1.8; C_1 = 0.05 ) ( (\gamma_s = 0.36) )</th>
<th>( \alpha = 1.4; C_1 = 0.1 ) ( (\gamma_s = 0.43) )</th>
<th>( \alpha = 0.6; C_1 = 0.1 ) ( (\gamma_s = 0.22) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up-stream conduit</td>
<td>42.9</td>
<td>30.3</td>
<td>46.4</td>
<td>39.3</td>
</tr>
<tr>
<td>Middle conduit</td>
<td>16.7</td>
<td>13.5</td>
<td>15.7</td>
<td>14.3</td>
</tr>
<tr>
<td>Outlet</td>
<td>18.2</td>
<td>9.7</td>
<td>14.0</td>
<td>12.4</td>
</tr>
</tbody>
</table>

3.4. Sensitivity of the results to the UM parameters

In this paper, the same UM parameters \( (\alpha = 1.8, C_1 = 0.1) \) have been used for the three events. They correspond to estimates commonly obtained in the literature when focusing the analysis on the rainy portions of the rainfall fields. In this section, we suggest to test the sensitivity of the results to the values of the UM parameters. To achieve this, the same methodology has been implemented for the 2011 event with various UM parameters sets. Results are summarized in Table 1. The value of the maximum probable singularity \( \gamma_s \) for each parameter set was added. It is a scale invariant estimate of the maximum probable value observable on a unique sample of the phenomenon, and has commonly been used to assess the extremes in the multifractal framework (Hubert et al., 1993; Douglas and Barros, 2003; Royer et al., 2008; Gires et al., 2011).

It appears that the values of the UM parameters have indeed an influence on the computed uncertainty. For example \( CV' \) for the outlet is almost twice decreased when \( C_1 = 0.05 \) rather than \( C_1 = 0.1 \) while \( \alpha \) is kept equal to 1.8. One can note that the uncertainty are lower for \( \alpha = 1.8 \), \( C_1 = 0.05 \) which corresponds to \( \gamma_s = 0.36 \) than for \( \alpha = 0.6 \), \( C_1 = 0.1 \) which corresponds to \( \gamma_s = 0.22 \). This result is not expected if only the notion of maximum singularity is used to assess the extremes of the rainfall fields. It simply means both UM parameters are needed to properly characterize the rainfall field and assess the uncertainty associated with small scale rainfall variability. From Table 1, it can be seen that \( C_1 \) has a stronger influence than \( \alpha \) on the estimated uncertainty, suggesting that efforts should be focused on its correct estimation.

4. Conclusion

Universal multifractals are used to quantify the uncertainty associated with small scale unmeasured (i.e. occurring at scales smaller than 1 km in space and 5 min in time) rainfall variability on the outputs of Multi-Hydro, a newly-developed fully distributed urban hydrologic/hydraulic numerical platform, and a standard semi-distributed 1D model implemented on the same 1.44 km² urban area located in Sevran, near Paris (France). Three rainfall events with return periods smaller than 1 year for durations of 1 h and 4 h are tested. First the models are roughly validated on both the main case study and an additional 0.5 km² urban catchment for which more measurements were available. Then the methodology basically consists in generating an ensemble of realistic downscaled rainfall fields and simulating the corresponding ensemble of hydrographs. This enables a quantification of the uncertainty. It appears that for the three rainfall events the uncertainty is rather elevated and cannot be neglected. For example \( CV' \) ranges between 15% and 21% at the outlet and between 37% and 61% for upstream conduits according to the event. Furthermore the uncertainty computed with the help of the fully distributed Multi-Hydro model is much greater than the one obtained with the 1D semi-distributed model, which means that fully distributed models would be needed to fully benefit from improved rainfall data. The sensitivity of the results to the two parameters used to downscale the rainfall field was tested and showed that special care should be dedicated to estimating them for applications. In this paper, only the sensitivity to rainfall resolution was tested. More generally similar work should be carried out on other common input fields such as the land use distribution, or the soil properties (especially the infiltration capacities). If obtained conclusions are similar, this would confirm that small scale phenomenon should to be taken into account much more carefully in urban hydrology. This points out that in terms of modelling the use of fully distributed models should be developed especially for applications dedicated to the RTC of sewer networks. The use of the notion of fractal dimension to characterize some features of the inputs of the model also suggests that the implementation of such tools, which are rather common in geophysics, should be developed in urban hydrology. In terms of rainfall, there is a need for higher resolution data in urban areas. To achieve this, the use of X-band radars which provide hectometric resolution would be highly beneficial. Further investigations with heavier rainfall events that generate urban pluvial flooding should also be performed to confirm this need for high resolution modelling.

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