Plasma Instabilities: Review

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• Instabilities: general (Abelian) picture
• Requirements for instabilities to make sense
• Growth rate, dependence on anisotropy
• How big do they grow?
• What do they do?
Suppose two streams of plasma collide:

becomes

What happens?
Magnetic field growth!

Consider the effects of a seed magnetic field $\hat{B} \cdot \hat{p} = 0$ and $\hat{k} \cdot \hat{p} = 0$

How do the particles deflect?
Positive charges:

No net $\rho$. Net current is induced as indicated.
Negative charges: same-sign current contribution

Induced $B$ adds to seed $B$. Exponential Weibel instability

Linearized analysis: $B$ grows until bending angles become large.
Note: particles in other directions are stabilizing

Sum of $J$ from two signs weakens seed magnetic field.

Isotropy: effects from different directions cancel!
What is $B$ field growth rate?

Force, velocity, deflection:

$$F = eB ; \quad \Delta v = \frac{tF}{p} = \frac{eBt}{p} ; \quad \Delta y = \frac{\Delta v t}{2} = \frac{eB}{2p} t^2 .$$

**concentration of charges:** $\sim \Delta y / \lambda_B$ or $k \Delta y \quad k$ the $B$-wavevector

$$J \sim e n_{\text{chg}} k \Delta y \sim e^2 B t^2 k \int \frac{d^3p}{(2\pi)^3 2p} f(p)$$

Define the combination

$$e^2 \int \frac{d^3p}{(2\pi)^3 p} f(p) \equiv m^2 \quad \text{Screening mass squared}$$
From last slide:

\[ J \sim (kB)m^2t^2. \]

Current must compete with field terms in Ampere's law:

\[ D \times B - D_tE = J \]

For current to \textit{really matter}, need \( J \sim D \times B \sim kB \).

This occurs when \( m^2t^2 \sim 1 \) or \( t \sim 1/m \).

Hence growth rate estimate: \( B \sim B_0e^{\gamma t}, \gamma \sim m \).

Picture is self-consistent \textbf{IF} particles stay in same-sign \( B \) field for time scales \( t\gamma > 1 \).
Abelian fields? NO! Nonabelian works too! But:

- Classical field approximation: need $\alpha \ll 1$
  
  I use $e^2, g^2, \alpha$ interchangeably

- Classical particle treatment: $\lambda_{p, \text{deBroglie}} \gg k_B^{-1}$ or $p \gg m, k_{\text{inst}}$. Allows HL approx.  Must be true in each direction!

Former: concepts make no sense at strong coupling (I think)
Latter: constraints on occupancies and level of anisotropy of excitations giving rise to instability
Dependence on Occupancy and Anisotropy

if typical momentum $p \sim Q$, $m^2 \sim \alpha Q^2 f$ ($f$ typical occupancy):

Higher Occupancy: Larger $m^2$ (red), $k_{\text{inst}}$ (black)
Dependence on Occupancy and Anisotropy

For high anisotropy, $m^2$ goes down, $k_{\text{inst}}$ goes up!

Smaller $m^2$: less filled phase space, fewer part. Larger $k_{\text{inst}}$: next!
High anisotropy and larger $k_{\text{inst}}$

Modes unstable whenever particles stay in same-sign $B$ for $t > 1/m$. Narrow momentum distrib: time can be longer!

Distance $1/m$

Dist. $\delta/m$

Time scale $t \sim \delta/k$ not $1/k$. So $k \sim m/\delta$ OK!
High anisotropy

\[ m^2 \sim \alpha^2 \int \frac{d^3p}{p} f(p) \sim \delta \alpha Q^2 f \quad \text{(less phase-space)} \]

But \( k_{\text{inst}} \sim m/\delta \sim \delta^{-1/2} \) larger.

Treatment \textit{inconsistent} if \( k_{\text{inst}} \sim m/\delta > \delta Q \sim p_z \)

Wave packet does not fit! Alternate analysis in this regime:

\[ k_{\text{inst}} \sim \sqrt{mQ} \]: more than \( \delta Q \), less than \( m/\delta \).
Weak anisotropy

What if almost-isotropic, with a few ($O(\epsilon)$) extras?

Isotropic part has no influence on what $k$’s unstable!

Repeat treatment using $\epsilon m^2$ in place of $m^2$.

$k_{\text{inst}} \sim \epsilon^{\frac{1}{2}} m$. But $\gamma \sim \epsilon^{\frac{3}{2}} m$. 
What limits $B$-field growth?

Many things \textit{can} do the job:

- Large angle change: $\Delta y < 1/k$ or $B < \delta m_p/g$ Robust

- Nielsen-Olesen instability: $B \lesssim k^2/g \sim m^2/(\delta^2 g)$ Robust

- Nonabelian Nonlinear Interactions: $B \lesssim k_\perp k_z/g \sim m^2/(\delta g)$ initial conditions?

Whichever works at smallest $B$-value is relevant.
Current stops building when particles bend too much.

\[ \Delta y > \frac{1}{k} \quad \Rightarrow \quad \Delta p > \delta p , \quad B > \delta m p / g \]
Nielsen-Olesen Instability

Uniform $B$ field: circular orbits, quantized $p_\perp$: ($\perp$ to $B$ that is)

\[ p_\perp^2 = (1 + 2n)eB . \]

Energies of allowed excitations $\textit{FOR SPIN-1}$:

\[ E^2 = p_\perp^2 + p_z^2 + 2e\vec{s} \cdot \vec{B} = p_z^2 + (2n + 1 \pm 2)eB \]

One mode has negative $E^2$; exp growth, $\gamma_{\text{N-O}} \sim \sqrt{gB}$

Uniform $B$ approx is OK if $p_\perp^2$ increments $2eB > k_{\text{inst}}^2$. So $k_{\text{inst}} < \sqrt{eB}$ to avoid instability
Nonabelian Nonlinearities

Suppose modes grow with many colors and $k$’s. One color acts to rotate $J$’s due to another color:

No Blue Current

No Blue Current
Requirement this happens: $A \sim \nabla$ in covariant deriv, $A \sim m / g$ or $B \sim m^2 / (g \delta)$. Gauge-invariant version:

Color randomization when Wilson loop shown has $\mathcal{O}(1)$ phase.
What do plasma instabilities do?

Main thing: angle-change.
$\Delta \theta \ll 1$ for self-consistency (we saw)
Many small independent “kicks”: describe with $\hat{q}$

$$\hat{q} \equiv \frac{dp_\perp^2}{dt} \sim \frac{(\Delta p)^2}{t_{coh}} = F^2t_{coh} \sim \alpha B^2t_{coh}$$

Now $B \sim m^2/g\delta$, $t_{coh} \sim 1/m$

$$\hat{q} \sim \frac{m^3}{\delta^2}$$

Thermal-like, $\mathcal{O}(1)$ anisotropic: $g^3T^3$ (elastic: $g^4T^3$)
Enhanced by $1/\delta^2$ when large anisotropy.
Where are plasma instabilities important?

Assume ONE typical momentum scale, eg, $Q_s$

$\delta = \alpha^d$

Anisotropy

$\Theta(p - Q_s) \Theta(|p_z - \alpha^d Q_s|)$

Occupancy-anisotropy plane.

$f = \alpha^{-c}$
Where are plasma instabilities important?

Consider feedback of instab. on hard modes, radiation, merging: Kukela Moore I

\[ d = \ln(\delta)/\ln(\alpha) \]
\[ c = -\ln(f)/\ln(\alpha) \]
\[ c = -1/3 \]

Anisotropy

Plasma Instabilities

Soft Particle Bath Forms

Scattering

Nielsen–Olesen Instabilities

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Thermal bath dominates:  \( t \sim \alpha_s^{-12/5} Q_s^{-1} \)
equilibration:  \( t \sim \alpha_s^{-5/2} Q_s^{-1} \).
Conclusions

• Plasma instabilities generic in anisotropic, $\alpha \ll 1$ plasmas

• Especially important in situations of high anisotropy

• More phenomena can limit growth in a nonabelian than in an abelian context

• Should play a pivotal role in the equilibration process in heavy ion collisions (in the toy case of $\alpha_s \ll 1$)

• To make quantitative predictions we need to understand the weak anisotropy case.