Extendability, analyticity, & reality

Refs: Wald §9 for definitions.

- Derivation of Kruskal extension is good story, but what did we really do? What of this story is physical?

- BHs are only going to get more strange, so goal today is to build our bullshit detectors...

Outline

- Extendability
- Analyticity
- BH formation
Extendibility

Defn: a spacetime \((M, g)\) is extendible if it is isometric to a proper subset of another manifold \((M', g')\). Then \((M', g')\) is an extension of \((M, g)\).

When is a ST extendible? Depends on your defn of ST:
- if you allow disconnected STs then every ST extendible!
- check b.c.'s, global hyperbolicity, \ldots\ (equiv. require global \text{I.V.P.})
- best criteria: geodesic completeness

Geodesic completeness

Recall geodesic eqn:

\[ u^\alpha \nabla_\alpha u^\beta = c u^\beta \]  \[ \text{, } u^\alpha \text{ tangent to curve} \]

In words, "tangent vector parallel prop. along itself points in same direction". Can parametrize curve so that \(c=0\) \text{ affine parametrization.}

Defn: a geodesic is \text{ incomplete} if it cannot be extended in at least 1 direction and has finite affine length.
**Def**: A spacetime is _geodesically incomplete_ if it contains an incomplete geodesic.

Geodesic incompleteness indicates that either:

1. ST is extendible
2. ST is singular (a defect of “singular”!)

This brings us to a conjecture:

**Strong cosmic censorship conjecture (Penrose)**

For geod. complete, asympt. flat initial data in vacuum, “generically” the maximal development is inextendible.

**Analyticity jor, are extensions unique?**

**No.** Assume \((M, g)\) maximal development of some partial Cauchy surface \(Z \subset M\). Then if \(\mathcal{E}\) an extension \(\mathcal{E}\) an \(\infty\) extensions.

**Eg., Schrs.**:

- Geodesics terminate at finite affine param. at \(r = 2M\).
One extension is that of Kruskal.
  - preserves $SO(3)$, dt symm.
  - preserves real analyticity of Schr. solution

But can consider other extensions:
  - add gravitywaves to region L, F
  - add another BH to L region!
  - can seed shock wave and destroy other regions
    (for fun see Shenker+Stamford papers)
  - can even glue on closed FRW cosmology

**Bottom line:** extensions aren't unique. What is unique is causal development. Anyway else is either adding initial data (initial data was not good, complete), or playing with timelike singularities (so as to extend beyond minimal causal development).

**Warning:** analytic extensions often advertised as making "minimal" assumptions...
Collapse geometries

In astrophysical settings a more realistic picture is

Stellar collapse

- $\not\Delta$ KVF
- Generically no $SO(3)$ symmetry, though not critical
- No white hole, I region, ER bridge
- Event horizon $\neq$ Killing horizon

That said, can model spherically symm. collapse w/ orig. Schwarzschild chart.