Bulk Reconstruction and Entropic Area Laws

Sebastian Fischetti
1805.08891 with N. Engelhardt

McGill University

October 15, 2018
AdS/CFT and Quantum Gravity

An ambitious question

The (semi)classical gravity we observe in our universe emerges from some more fundamental quantum theory - how?
AdS/CFT and Quantum Gravity

An ambitious question

The (semi)classical gravity we observe in our universe emerges from some more fundamental quantum theory - how?

- Hard to even begin to answer because we don’t have a full formulation of such a theory!
- In context of string theory, AdS/CFT gives us a nonperturbative formulation of a theory of quantum gravity
- But this definition is very indirect - need dictionary to reformulate boundary theory into gravitational language
AdS/CFT and Quantum Gravity

An ambitious question

The (semi)classical gravity we observe in our universe emerges from some more fundamental quantum theory - how?

- Hard to even begin to answer because we don’t have a full formulation of such a theory!
  - In context of string theory, AdS/CFT gives us a nonperturbative formulation of a theory of quantum gravity
  - But this definition is very indirect - need dictionary to reformulate boundary theory into gravitational language

A slightly less vague question

In AdS/CFT, when and how does (semi)classical gravity emerge from boundary field theory?
AdS/CFT and Quantum Gravity

A few related questions:

- What does it mean for a field theory to be holographic? When is a field theory holographic? [Heemskerk, Penedones, Polchinski, Sully]

- Given a holographic field theory, what are the dynamics of the dual gravitational theory? [Lashkari, McDermott, Van Raamsdonk, ...]
A few related questions:

- What does it mean for a field theory to be holographic? When is a field theory holographic? [Heemskerk, Penedones, Polchinski, Sully]

- Given a holographic field theory, what are the dynamics of the dual gravitational theory? [Lashkari, McDermott, Van Raamsdonk, ...]

- Given a state of a holographic field theory, is there a semiclassical dual geometry? If so, how is it (or any of its properties) obtained from the boundary state? [Van Raamsdonk; Czech, Lamprou; Engelhardt, Horowitz; ...]

- Given a dual classical geometry, how does semiclassical local physics on this geometry emerge from the boundary? [...]
AdS/CFT and Quantum Gravity

A few related questions:

- What does it mean for a field theory to be holographic? When is a field theory holographic? [Heemskerk, Penedones, Polchinski, Sully]

- Given a holographic field theory, what are the dynamics of the dual gravitational theory? [Lashkari, McDermott, Van Raamsdonk, ...]

- Given a state of a holographic field theory, is there a semiclassical dual geometry? If so, how is it (or any of its properties) obtained from the boundary state? [Van Raamsdonk; Czech, Lamprou; Engelhardt, Horowitz; ...]

- Given a dual classical geometry, how does semiclassical local physics on this geometry emerge from the boundary? [...]

Sebastian Fischetti
McGill University
Recovering Bulk Operators

In pure AdS, local field operators can be expressed in terms of local boundary operators by integrating against a kernel [Hamilton, Kabat, Lifschytz, Lowe]:

\[ \phi(X) = \int_{D \subset \partial M} d^d x \, K(X|x) \mathcal{O}(x) \]
Recovering Bulk Operators

- In pure AdS, local field operators can be expressed in terms of local boundary operators by integrating against a kernel [Hamilton, Kabat, Lifschytz, Lowe]:

\[ \phi(X) = \int_{D \subset \partial M} d^dx K(X|x) \mathcal{O}(x) \]

- Kernel may be taken to have support on different boundary regions \( D \)
Recovering Bulk Operators

- In pure AdS, local field operators can be expressed in terms of local boundary operators by integrating against a kernel [Hamilton, Kabat, Lifschytz, Lowe]:

\[ \phi(X) = \int_{D \subset \partial M} d^d x \ K(X|x) \mathcal{O}(x) \]

- Kernel may be taken to have support on different boundary regions \( D \)
Recovering Bulk Operators

- In pure AdS, local field operators can be expressed in terms of local boundary operators by integrating against a kernel [Hamilton, Kabat, Lifschytz, Lowe]:

\[ \phi(X) = \int_{D \subset \partial M} d^d x \ K(X|x) \mathcal{O}(x) \]

- Kernel may be taken to have support on different boundary regions \( D \)

- Subregion/subregion duality: a given boundary diamond \( D \) can reconstruct local operators in some subregion of the bulk
Subregion/Subregion Duality

- Causal argument suggests that can only recover operators causally separated from $D$
Subregion/Subregion Duality

- Causal argument suggests that can only recover operators causally separated from $D$
- Too naïve: RT/HRT say that

$$S[D] = - \text{Tr}(\rho_D \ln \rho_D) = \frac{\text{Area}[\Sigma_D]}{4G_N \hbar}$$

with $\Sigma_D$ minimal-area extremal surface homologous to $D$
Subregion/Subregion Duality

- Causal argument suggests that can only recover operators causally separated from $D$
- Too naïve: RT/HRT say that
  \[
  S[D] = - \text{Tr}(\rho_D \ln \rho_D) = \frac{\text{Area}[\Sigma_D]}{4G_N \hbar}
  \]
  with $\Sigma_D$ minimal-area extremal surface homologous to $D$
- $\Sigma_D$ generically is spacelike to $D$, so entanglement entropy probes deeper into bulk than causal intuition implies: non-local operators can reconstruct deeper

[Czech, Karczmarek, Nogueira, Raamsdonk]
Subregion/Subregion Duality

- Causal argument suggests that can only recover operators causally separated from $D$
- Too naïve: RT/HRT say that

$$S[D] = -\text{Tr}(\rho_D \ln \rho_D) = \frac{\text{Area}[\Sigma_D]}{4G_N\hbar}$$

with $\Sigma_D$ minimal-area extremal surface homologous to $D$
- $\Sigma_D$ generically is spacelike to $D$, so entanglement entropy probes deeper into bulk than causal intuition implies: non-local operators can reconstruct deeper
  - [Czech, Karczmarek, Nogueira, Raamsdonk]
- Region that can be reconstructed is the entanglement wedge $W_E[D]$
Quantum Error Correction

- Proof of entanglement wedge reconstruction comes from combining [Jafferis, Lewkowykz, Maldacena, Suh] and quantum error correction [Almheiri, Dong, Harlow]

- There’s redundancy in which boundary regions a local bulk operator can have support on:
Quantum Error Correction

- Proof of entanglement wedge reconstruction comes from combining [Jafferis, Lewkowykz, Maldacena, Suh] and quantum error correction [Almheiri, Dong, Harlow]
- There’s redundancy in which boundary regions a local bulk operator can have support on:
  \[ \Sigma_{D_1} \]
  \[ \bullet X \]
  \[ \Sigma_{D_2} \]
  \[ \bullet X \]

- The classical background identifies a subspace of states (the code subspace), and the different reconstructions are redundant only in this subspace
- Can then prove that any operator in \( W_E[D] \) can be reconstructed (on code subspace) from \( D \) [Dong, Harlow, Wall; Faulkner, Lewkowycz]
What About the Background?

- But given just the boundary state, don’t know what the corresponding code subspace is (or even if there is one)
- This is precisely the question of the emergence of a classical spacetime - lots of interesting physics!
What About the Background?

- But given just the boundary state, don’t know what the corresponding code subspace is (or even if there is one)
- This is precisely the question of the emergence of a classical spacetime - lots of interesting physics!
- Can try to reconstruct the full geometry, but this is hard. Partial progress:
  - Near boundary can just use Fefferman-Graham expansion
  - Hole-ography can do a little in 3D [Czech, Lamprou], though can’t go too deep [Engelhardt, SF]
  - Can get causal structure from singularities of correlators [Engelhardt, Horowitz; Engelhardt, SF], but again can’t go past causal wedge
  - See later in talk (if time permits)
What About the Background?

- But given just the boundary state, don’t know what the corresponding code subspace is (or even if there is one)
- This is precisely the question of the emergence of a classical spacetime - lots of interesting physics!
- Can try to reconstruct the full geometry, but this is hard. Partial progress:
  - Near boundary can just use Fefferman-Graham expansion
  - Hole-ography can do a little in 3D [Czech, Lamprou], though can’t go too deep [Engelhardt, SF]
  - Can get causal structure from singularities of correlators [Engelhardt, Horowitz; Engelhardt, SF], but again can’t go past causal wedge
  - See later in talk (if time permits)
- Instead, try recovering gravitationally interesting geometric features: area laws!
Why Area Laws?

- Properties of classical spacetimes, but connected to gravitational thermodynamics - presumably emerge from some coarse-graining mechanism.

- Have some understanding of this for Bekenstein-Hawking entropy of BPS black holes [Strominger, Vafa]

- For dynamical black holes, less is known: interesting candidates are event horizon (globally defined) and holographic screens/apparent horizons (locally defined).
Why Area Laws?

Have understanding of area law along apparent horizons (spacelike part of $H^+$) emerging from a coarse-graining mechanism, though boundary interpretation not completely understood [Engelhardt, Wall].

Still no entropic explanation for dynamical event horizons $H^+$ or mixed-signature holographic screens $\mathcal{H}^+$.

Try to come up with a more universal microscopic understanding.
Why Area Laws?

- Have understanding of area law along apparent horizons (spacelike part of $H^+$) emerging from a coarse-graining mechanism, though boundary interpretation not completely understood [Engelhardt, Wall]

- Still no entropic explanation for dynamical event horizons $\mathcal{H}^+$ or mixed-signature holographic screens $H^+$
Why Area Laws?

- Have understanding of area law along apparent horizons (spacelike part of $H^+$) emerging from a coarse-graining mechanism, though boundary interpretation not completely understood [Engelhardt, Wall]

- Still no entropic explanation for dynamical event horizons $\mathcal{H}^+$ or mixed-signature holographic screens $H^+$

- Try to come up with a more universal microscopic understanding
Coarse-Graining

- Coarse-graining is supposed to remove gravitational UV degrees of freedom.
- By UV/IR correspondence, UV of bulk theory corresponds to IR of boundary, so let’s introduce a prescription for discarding IR data in the boundary.
Coarse-Graining

- Coarse-graining is supposed to remove gravitational UV degrees of freedom
- By UV/IR correspondence, UV of bulk theory corresponds to IR of boundary, so let’s introduce a prescription for discarding IR data in the boundary
- Consider a continuous family $F = \{D_\lambda\}$ of causal diamonds in some (arbitrary) QFT:
Coarse-Graining

- Coarse-graining is supposed to remove gravitational UV degrees of freedom
- By UV/IR correspondence, UV of bulk theory corresponds to IR of boundary, so let’s introduce a prescription for discarding IR data in the boundary
- Consider a continuous family $F = \{D_\lambda\}$ of causal diamonds in some (arbitrary) QFT:

\[
\begin{array}{cccccc}
\cdots & D_\lambda_1 & & D_\lambda_2 & \cdots & D_\lambda_n \cdots \\
\end{array}
\]

- Restricting a full state $\rho$ to the set $\rho_F = \{\rho_{D_\lambda}\}$ of reduced states removes knowledge of correlations between points that aren’t contained in any single diamond: $\rho \rightarrow \rho_F$ is coarse-graining
Bulk Picture

- If the QFT state has a geometric bulk dual, subregion/subregion duality tells us what this corresponds to in the bulk
Bulk Picture

- If the QFT state has a geometric bulk dual, subregion/subregion duality tells us what this corresponds to in the bulk.
Bulk Picture

- If the QFT state has a geometric bulk dual, subregion/subregion duality tells us what this corresponds to in the bulk.

- A “deep bulk” region is completely unrecoverable, but can recover local operators near the asymptotic region (related to [Nomura, Rath, Salzetta]).

- Consistent with rough interpretation of e.g. BH entropy as arising from ignorance of interior of black hole.
Differential Entropy and Hole-ography

- Now work in (2+1)-d bulk
- From family of regions $F$ can define differential entropy:

$$S_{\text{diff}}[F] = \lim_{n \to \infty} \sum_{i=1}^{n} (S[D_i] - S[D_i \cap D_{i+1}])$$

$S_{\text{diff}}[F]$ computes the length of some curve(s) $\sigma_F$ in the bulk constructed from the entanglement wedges of $\{D_{\lambda}\}$. [Balasubramanian, Chowdhury, Czech, de Boer, Heller; Headrick, Myers, Wien]

No general physical interpretation of $S_{\text{diff}}[F]$, but partial one is as the cost of a constrained state swapping protocol [Czech, Hayden, Lashkari, Swingle].

Sebastian Fischetti

Bulk Reconstruction and Entropic Area Laws
Now work in (2+1)-d bulk

From family of regions $F$ can define differential entropy:

$$S_{\text{diff}}[F] = \lim_{n \to \infty} \sum_{i=1}^{n} (S[D_i] - S[D_i \cap D_{i+1}])$$

$S_{\text{diff}}[F]$ computes the length of some curve(s) $\sigma_F$ in the bulk constructed from the entanglement wedges of $\{D_\lambda\}$

[Balasubramanian, Chowdhury, Czech, de Boer, Heller; Headrick, Myers, Wien]:

$$S_{\text{diff}}[F] = \frac{\text{Length}[\sigma_F]}{4G_N \hbar}$$

No general physical interpretation of $S_{\text{diff}}[F]$, but partial one is as the cost of a constrained state swapping protocol [Czech, Hayden, Lashkari, Swingle]
Differential Entropy and Hole-ography

\[ \partial (\bigcup \lambda W_E[D\lambda]) \]
Monotonicity from SSA

- What happens as we further coarse-grain $F = \{D_\lambda\}$ to $\hat{F} = \{\hat{D}_\lambda\}$ with $\hat{D}_\lambda \subset D_\lambda$? (“Weakening the QECC”)

Recall strong subadditivity of entanglement entropy:


Implies irreversibility under removal of subsystems: in terms of mutual information,

$$I(A|B) \leq I(A|BC)$$
Monotonicity from SSA

- What happens as we further coarse-grain $F = \{D_\lambda\}$ to $\hat{F} = \{\hat{D}_\lambda\}$ with $\hat{D}_\lambda \subset D_\lambda$? (“Weakening the QECC”)

Recall strong subadditivity of entanglement entropy:


Implies irreversibility under removal of subsystems: in terms of mutual information, $I(A|B) \leq I(A|BC)$
Monotonicity from SSA

\[
\Delta S_{\hat{D}_i} - \Delta S_{\hat{D}_i \cap \hat{D}_{i+1}} = S_{A_i B_i C_i} - S_{A_i B_i C_i} - S_{B_i} \geq 0
\]

\[
S_{\text{diff}}[\hat{F}] \geq S_{\text{diff}}[F] \Rightarrow \text{area law}
\]
Monotonicity from SSA

- Applied to $F$ and $\hat{F}$,

$$\left( S[\hat{D}_i] - S[\hat{D}_i \cap \hat{D}_{i+1}] \right) - \left( S[D_i] - S[D_i \cap D_{i+1}] \right)$$

$$= S[A_i B_i] + S[B_i C_i] - S[A_i B_i C_i] - S[B_i] \geq 0$$
Monotonicity from SSA

- Applied to $F$ and $\hat{F}$,

$$
\left( S[\hat{D}_i] - S[\hat{D}_i \cap \hat{D}_{i+1}] \right) - \left( S[D_i] - S[D_i \cap D_{i+1}] \right)
= S[A_i B_i] + S[B_i C_i] - S[A_i B_i C_i] - S[B_i] \geq 0
$$

- $S_{\text{diff}}[\hat{F}] \geq S_{\text{diff}}[F] \Rightarrow \text{area law}$
Area Laws

Take-home Message

In (2+1)-bulk dimensions, we obtain a family of area laws which are a precise manifestation of strong subadditivity! Coarse-graining comes from removing long-distance correlators on the boundary*. 

* Caveat: interpretation in terms of coarse-graining isn't quite correct due to vacuum rigidity; if \( \rho \) is vacuum, \( \rho_F \) is sufficient to tell you're in vacuum.
Area Laws

Take-home Message

In (2+1)-bulk dimensions, we obtain a family of area laws which are a precise manifestation of strong subadditivity! Coarse-graining comes from removing long-distance correlators on the boundary.*

- Proof is essentially the same as the Casini-Huerta entropic $c$-theorem, except no need for Poincaré invariant states
- Like $c$-theorem, the interpretation of the monotonicity is clear even if the interpretation of the thing that’s monotonic ($S_{\text{diff}}$) is not
Area Laws

Take-home Message

In (2+1)-bulk dimensions, we obtain a family of area laws which are a precise manifestation of strong subadditivity! Coarse-graining comes from removing long-distance correlators on the boundary*.

- Proof is essentially the same as the Casini-Huerta entropic $c$-theorem, except no need for Poincaré invariant states
- Like $c$-theorem, the interpretation of the monotonicity is clear even if the interpretation of the thing that’s monotonic ($S_{\text{diff}}$) is not

*Caveat: interpretation in terms of coarse-graining isn’t quite correct due to vacuum rigidity; if $\rho$ is vacuum, $\rho_F$ is sufficient to tell you’re in vacuum
Some Examples

Null; include Hawking area law for a simple causal horizon
Some Examples

Spacelike
Some Examples

Mixed-signature; signature change similar to holographic screens
Higher Dimensions

- In higher dimensions, for appropriate choices of the family $F$ it's still possible to construct surfaces with monotonic area from the $W_E[D_\lambda]$
Higher Dimensions

- In higher dimensions, for appropriate choices of the family $F$ it’s still possible to construct surfaces with monotonic area from the $W_E[D_\lambda]$

- But generalization of $S_{\text{diff}}$ to higher dimensions is unknown, so lose the precise connection to SSA

- Hints from Casini-Huerta: there are also entropic $F$- and $a$-theorems, so why not try constructing higher-d “differential entropy” by generalizing those?
Higher Dimensions

- In higher dimensions, for appropriate choices of the family $F$ it’s still possible to construct surfaces with monotonic area from the $W_E[D_\lambda]$

- But generalization of $S_{\text{diff}}$ to higher dimensions is unknown, so lose the precise connection to SSA

- Hints from Casini-Huerta: there are also entropic $F$- and $a$-theorems, so why not try constructing higher-d “differential entropy” by generalizing those?

- Future work!
Quantum Generalization

- Are our area laws really saying something about gravitational thermodynamics or just artifacts of the classical limit?
Quantum Generalization

- Are our area laws really saying something about gravitational thermodynamics or just artifacts of the classical limit?

- With quantum corrections, HRT formula gets modified [Faulkner, Lewkowycz, Maldacena; Engelhardt, Wall; Dong, Lewkowycz]:

\[ S[D] = S_{\text{gen}}[\Sigma_D] = \frac{\text{Area}[\Sigma_D]}{4G_N \hbar} + S_{\text{out}}[\Sigma_D] \]

with \( \Sigma_D \) quantum extremal surface (extremizes \( S_{\text{gen}}[\Sigma_D] \))

- Then can generalize the general classical results to show that for appropriate choice of \( F \), can construct bulk surfaces \( \sigma_F \) (from \( \Sigma_{D,\lambda} \)) such that \( S_{\text{gen}}[\sigma_{\hat{F}}] \geq S_{\text{gen}}[\sigma_F] \)
Quantum Generalization

- Are our area laws really saying something about gravitational thermodynamics or just artifacts of the classical limit?

- With quantum corrections, HRT formula gets modified [Faulkner, Lewkowycz, Maldacena; Engelhardt, Wall; Dong, Lewkowycz]:

\[ S[D] = S_{\text{gen}}[\Sigma_D] = \frac{\text{Area}[\Sigma_D]}{4G_N \hbar} + S_{\text{out}}[\Sigma_D] \]

with \( \Sigma_D \) quantum extremal surface (extremizes \( S_{\text{gen}}[\Sigma_D] \))

- Then can generalize the general classical results to show that for appropriate choice of \( F \), can construct bulk surfaces \( \sigma_F \) (from \( \Sigma_{D,\lambda} \)) such that \( S_{\text{gen}}[\sigma\hat{F}] \geq S_{\text{gen}}[\sigma_F] \)

- But a quantum generalization of the precise connection using SSA is still lacking, and would presumably include something like differential entropy of bulk
Bonus Aside: Metric Reconstruction

- Subregion/subregion duality suggests that $W_E[D]$, including metric, should be recoverable from $D$.
- Operators in $W_E[D]$ are recovered from modular flow; what data in $D$ is needed to recover metric?
Bonus Aside: Metric Reconstruction

- Subregion/subregion duality suggests that $W_E[D]$, *including metric*, should be recoverable from $D$.
- Operators in $W_E[D]$ are recovered from modular flow; what data in $D$ is needed to recover metric?
- Natural guess is to use entanglement entropy.
Bonus Aside: Metric Reconstruction

- Subregion/subregion duality suggests that $W_E[D]$, including metric, should be recoverable from $D$
- Operators in $W_E[D]$ are recovered from modular flow; what data in $D$ is needed to recover metric?
- Natural guess is to use entanglement entropy
- Partial progress in (2+1)-d using hole-ography made in [Czech, Lamprou], but can’t reach strong-gravity regions [Engelhardt, SF]
- Boundary rigidity problem: given areas of boundary-anchored extremal surfaces, is metric unique?
Subregion/subregion duality suggests that \( W_E[D] \), including metric, should be recoverable from \( D \).

Operators in \( W_E[D] \) are recovered from modular flow; what data in \( D \) is needed to recover metric?

Natural guess is to use entanglement entropy.

Partial progress in (2+1)-d using hole-ography made in [Czech, Lamprintou], but can’t reach strong-gravity regions [Engelhardt, SF].

Boundary rigidity problem: given areas of boundary-anchored extremal surfaces, is metric unique?

Work in progress with N. Bao, C. Cao, C. Keeler: using techniques from [Alexakis, Balehowsky, Nachman], for a (3+1) bulk, seems that knowledge of areas of arbitrary perturbations of a foliation of boundary-anchored extremal surface is sufficient to guarantee uniqueness of metric (still dotting “i”s and crossing “t”s, though!)
Open Questions

- In (2+1) bulk, have derived a class of area laws which correspond precisely to SSA in boundary theory.
- In certain contexts, these laws match the Hawking area law for event horizons, and they show the same mixed-signature behavior of holographic screens - suggests a universal microscopic mechanism.
Open Questions

- In (2+1) bulk, have derived a class of area laws which correspond precisely to SSA in boundary theory.
- In certain contexts, these laws match the Hawking area law for event horizons, and they show the same mixed-signature behavior of holographic screens - suggests a universal microscopic mechanism.
- Can we generalize the precise connection to SSA to higher dimensions?
Open Questions

- In (2+1) bulk, have derived a class of area laws which correspond precisely to SSA in boundary theory.
- In certain contexts, these laws match the Hawking area law for event horizons, and they show the same mixed-signature behavior of holographic screens - suggests a universal microscopic mechanism.
- Can we generalize the precise connection to SSA to higher dimensions?
- What’s the correct quantum generalization? Related: what’s the precise coarse-graining picture (which addresses vacuum rigidity issue)?
Open Questions

- In (2+1) bulk, have derived a class of area laws which correspond precisely to SSA in boundary theory.
- In certain contexts, these laws match the Hawking area law for event horizons, and they show the same mixed-signature behavior of holographic screens - suggests a universal microscopic mechanism.
- Can we generalize the precise connection to SSA to higher dimensions?
- What’s the correct quantum generalization? Related: what’s the precise coarse-graining picture (which addresses vacuum rigidity issue)?
- How far into the bulk can they reach? Can they always reproduce the familiar area laws, or only sometimes?