Problem Set 07

**Note:** The problem set is due October 24 by midnight. Please return directly to me in my office Rutherford 321. If I’m not there slip your assignment below my door.

1. Following are a collection of short and medium-long answer questions. As usual keep your answers to the point.

(a) An inverted hemispherical bowl of radius $R$ carries an uniform surface charge density $\sigma$. Show that the potential difference between the “north pole” and the center is given by:

$$V = \frac{R\sigma \sqrt{2} - 1}{2} \approx \frac{0.207 R\sigma}{\epsilon_0}$$

(b) Two point charges of masses $m_1$ and $m_2$ and charges $Q_1$ and $Q_2$ respectively are suspended at a common point by two threads of negligible mass and length $l$. What is the angle between the two threads at equilibrium? If the masses and charges are same, what is the equilibrium angle?

A slight variant of the above problem is to take three identical small spheres of mass $m$ suspended from a common point by threads of negligible mass and equal lengths $l$. A charge $Q$ is divided equally among the spheres, and they come to equilibrium at the corners of a horizontal equilateral triangle whose sides are $\sqrt{3}d$. If $g$ is the acceleration due to gravity, show that:

$$Q = \sqrt{\frac{36\sqrt{3}\epsilon_0 m g d^3}{\sqrt{l^2 - d^2}}} \approx 7.8964 \sqrt{\frac{\epsilon_0 m g d^3}{\sqrt{l^2 - d^2}}}$$

(c) Show that the electric field at point $(0, 0, h)$ due to the rectangle described by $-a \leq x \leq a$, $-b \leq y \leq b$, $z = 0$ carrying a uniform charge density $\rho$ is given by:

$$E = \frac{\rho}{\pi \epsilon_0} \tan^{-1} \left[ \frac{ab}{h\sqrt{a^2 + b^2 + h^2}} \right] \hat{z}$$

On the other hand, if I replace the rectangle by the plane $x + 2y = 5$ and $\rho = 6C/m^2$, determine $E$ at $(-1, 0, 1)$.

(d) Show that in 2+1 spacetime dimensions, the potential at any point in space is given by the average of the potential over a ring surrounding the point. This is called the average law, as we discussed in the class. Argue the average law for potential in 1+1 dimensions. An immediate consequence of the average law is that the potential has no local maxima or minima. Using these two concepts, prove the Earnshaw’s theorem, namely: a charged
particle cannot be held in stable equilibrium by electrostatic forces along. How is this result consistent with Q3 of assignment 5?

(e) We would like to construct a charged sphere of radius $R$ with a charge density that grows linearly with the radius. How much energy do we have to spend to build this configuration? On the other hand, if the charge density of the sphere is uniform, how much energy do we need to spend to build the charged sphere? Determine also the net force that the southern hemisphere of a uniformly charged sphere exerts on the northern hemisphere.

(f) A point charge $Q$ is located at point $(0,-4,0)$, while a 10 $C$ charge is uniformly distributed along a semicircular ring satisfying:

$$x^2 + y^2 = 4, \quad y > 0$$

Find the value of $Q$ such that the electric field at the origin of the coordinate system vanishes.

(g) Two spheres, each of radius $R$ and carrying uniform charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap. This is shown in the following figure:

Call the vector from the positive center to the negative center as $d$. Show that the field in the region of overlap is a constant and find its value.

2. Solve problems 3.6, 3.7 and 3.9 from Griffiths’ textbook. For the last problem, use the solution to problem 2.47 in the textbook and as done in the previous assignment.