GR is the theory of classical gravity.

Basic Idea: Forces are described by fields

e.g. 1. Newtonian GR field $\Phi$

2. EM $\vec{E} = q \vec{B}$

A field theory has 2 parts...

1. E&M which determines the field
   in terms of sources. "Field eqn"
   
   $\nabla^2 \Phi = (4\pi G) \rho$

   $\nabla \cdot \vec{E} = \rho$ ... etc.

2. Force law: determine the motion of
   objects in the presence of the field!

   $\vec{F} = m\dot{\vec{a}} = m \dot{\nabla} \Phi$

   $\Rightarrow = q (\vec{E} + \vec{v} \times \vec{B})$
So far we have taken the fields to be function of \((t, x)\) i.e., \(\phi = \phi(t, x)\),
\[\vec{E} = \vec{E}(t, x)\]

The force law tells us how motion will differ from a straight line in the field,
\(\vec{a} = 0 \Rightarrow \text{line}.

GR: Gravity is not due to a field which is a function of \((t, x)\) but rather to a feature of \(ST\) itself.

\(\Phi \Rightarrow \text{A "metric tensor" } g_{\mu\nu} \text{ which describes curvature of } ST\)

The field eqn determine the curvature of \(ST\) in terms of sources
\[R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}\]

"Einstein Eqn"
\[\nabla^2 \phi = 4\pi G \rho\]

The Force law is the geodesic eqn.
\[
\ddot{x}^M + \Gamma^M_{np} \dot{x}^n \dot{x}^p = 0
\]
describes how objects move when ST is curved.

Objects move in "straight lines" in curved ST.

This notion completely characterizes classical gravity.
Space-time is the set of all \((t, \vec{x})\).

A "point" in space-time is an "event".

\(ST\) is a set of "events" which can be parametrized by, e.g., cartesian or polar coords. in a smooth way.

\((t, \vec{x})\) or \((t, r, \Theta, \phi)\).

General Covariance: physics should be indep. of the choice of coord. system.

In relativistic physics, for two events \((t_1, \vec{x}_1)\) and \((t_2, \vec{x}_2)\):

\[\Delta t = t_2 - t_1,\]

\[\Delta x = \sqrt{(\vec{x}_2 - \vec{x}_1) \cdot (\vec{x}_2 - \vec{x}_1)}.\]

The two quantities \(\Delta t\) and \(\Delta x\) make sense in relativistic physics.
In SR, $\Delta t + \Delta x$ do not make sense independently. There is no independent notion of the time separation or spatial separation between 2 events...

These notions depend on which reference frame we use.

In SR, there is one notion which does make sense independently:

"Invariant Interval" $\Delta s$

$$\Delta s^2 = -c^2 \Delta t^2 + \Delta x^2$$

$$= -\Delta t^2 + \Delta x^2$$

We use units $c = 3 \times 10^8 \text{ m/s}$

$$= 1 \text{ light-second/second}$$

Claim: All of SR is just the statement that for two events $(t_1, x_1)$ and $(t_2, x_2)$, the time measured by an observer moving at most velocity between these events...
\[ i s \quad (\Delta \tau)^2 = - (\Delta s)^2 \]