Physics 610, Problem Set 11
due: Friday, December 4 at 3:30pm

Please place your completed problem sets in the “Physics 610” box in the physics department mailroom (Rutherford 103b) before the due date. Please do not leave them in my mailbox. You are encouraged to discuss these problems with your colleagues, but you must write up your own solutions; the solutions you hand in should reflect your own work and understanding. Late problem sets will be penalized 10% per day late, unless an extension has been obtained from me or the TA before the due date.

Announcements:

• The final problem set (problem set 12) will be handed out next week and will be due on Friday, December 12.

• I will be out of town on Thursday, December 3, but class will be held as usual; there will be a secret surprise guest lecturer.

• There will be an extra make-up lecture on Tuesday, December 8 at 3:30pm. Unless you hear from me otherwise, it will be in the usual classroom.

1. Trace Identities

Prove the following trace identities for gamma matrices, which we used when computing spin polarization sums. In many cases you should not need to use explicit expressions for gamma matrices, only the Clifford algebra.

(a) $\text{Tr}(\gamma^\mu) = 0$

(b) $\text{Tr}(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}$

(c) $\text{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho) = 0$

(d) $\text{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$

2. Bhaba Scattering

Consider the process of electron-positron ($e^+e^- \rightarrow e^+e^-$) scattering in QED.

(a) Compute the amplitude for this process by summing two Feynman diagrams. Your answer will depend on the polarizations of the initial and final electrons and positrons. Note that, unlike the $e^+e^- \rightarrow \mu^+\mu^-$ process considered in class, your answer will be the sum of two Feynman diagrams. The overall sign is not too important, but make sure to get the relative sign between these two Feynman diagrams correct!
(b) Imagine that we do not measure the polarizations of the initial or final states. Compute the scattering probability $\frac{1}{4} \sum |M|^2$, where we average over initial polarizations and sum over final polarizations. Write your answer as a function of the momenta of the incoming and outgoing particles. You may make the approximation where $E_{cm} >> m$, so that we can just ignore the electron mass.

(c) Now compute the differential cross section in centre of mass frame, and write it as a function of the Mandelstam variables. You should find

$$\frac{d\sigma}{d\Omega} = \frac{\pi \alpha^2}{s} \left( u^2 \left( \frac{1}{s} + \frac{1}{t} \right)^2 + \left( \frac{t}{s} \right)^2 + \left( \frac{s}{t} \right)^2 \right)$$

(d) Write this formula as a function of $E_{cm}$ and $\theta$ and plot it as a function of $\theta$ for fixed $E_{cm}$. Note that the result diverges at $\theta \to 0$. Can you explain why?

3. Muon Decay

In class we introduced the Feynman rules for QED, and generalized the theory to include muons. Muons are just like electrons, but more massive, so our theory of QED for electrons and muons has two Dirac fermion fields $\psi_e$ and $\psi_\mu$, with masses $m_e$ and $m_\mu$ and Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_e (i\not{D} - m_e) \psi_e + \bar{\psi}_\mu (i\not{D} - m_\mu) \psi_\mu,$$

where $\not{D} = \gamma^\mu (\partial_\mu - ieA_\mu)$ as usual.

The muon is very heavy compared to the electron ($m_\mu >> m_e$) so you might think that this theory would allow a muon to decay. Indeed, one of the most famous properties of the muon is that it is unstable. Show, however, that in this simple model the muon is stable. (There are many different ways to show this).

In order to study the decay of the muon we will have to introduce more complicated interactions (involving the weak nuclear force) that allow muons to decay into electrons and neutrinos.