LECTURE 19

Force and Pressure

Introduction

By now you are no doubt aware that I like to open a lecture with some demonstration that has results that you would not predict unless you know some physics. The demonstration for this lecture is set up by stringing a long cable across the lecture platform. One end is tied to a rigid support point. The other is tied to a sturdy piece of wood. You can see that this piece of wood is sturdy from my swinging from it with all my weight.

I then go to the center of the cable and pull it downward. With very little effort there is a loud crack as the wood breaks.

The question is from where did this force come? It was certainly the force of my hand that caused it but the force of my hand was clearly much less than the force it caused. How could this be?

This amplification of force by geometry is a very important phenomenon in nature. It is of particular importance in the invention of tools by humans. In medicine one of the routine applications of the phenomenon is in physiotherapy where devices have to be arranged to put specific forces on parts of the human body to correct disorders. This lecture is an introduction to such manipulation of forces. It will include the concept of pressure and how it relates to such forces.

Physics Definitions of Force and Pressure

The words "force" and "pressure" have many shades of meaning in English. However, they are generally associated with agents of change. Thus things try to force you to change, or apply pressure on you to change. Alternatively, they are often thought of as things to be resisted if you do not want to change. You withstand the force, or you cope with the pressure.

In physics, we again take a very restricted meaning for these words. In the case of force it is that of an agent which causes a mass to move. The most common such force is that of the weight of objects. This weight is the force that causes an object to fall to Earth.

To prevent an object from falling, the force of its weight must be resisted. When you are holding an object this force is the force you must apply by your hand. For example, if you are holding a mass in the palm of your hand, that force is the upward thrust of your hand against the bottom of the mass.

To hold the object steady you must, of course, apply exactly the same force as the weight of the object. For example, if the weight was one kilogram and you applied an upward force of 1100 grams, the mass would rise. If you applied only a force of 900 grams, the mass would fall. You must apply exactly 1000 grams. In fact, it is one of the remarkable things about the human nervous system that it can generate exactly the right force to keep things like the mass in your hand exactly where you want it to be.

However, suppose you had several different 1-kilogram objects with different diameter bottoms. Suppose one had a nice flat diameter of 6 cm while another had a stub on its bottom with
a diameter of only 1 cm and a third had a sharp
tip of only 1 mm diameter.

Each of these masses would cause your hand
to apply an upward force of exactly one kilogram,
but they would certainly feel different. The
discomfort caused by the middle object, and the
severe pain caused by the pointy-bottomed object
would be due to the pressure caused by the
weight of the objects where the pressure is related
to the way the weight is spread out over your
hand. If it is spread out over a large area then the
pressure is said to be low. If the weight is
concentrated in a small area the pressure is said to
be high.

Again, in physics we must have exact
definitions so that we can get numbers from
measurements that can be studied for possible
mathematical relationships. Here the definition is
straightforward. It is simply that the pressure is
the force divided by the area over which that force
is spread:

\[ p = \frac{F}{A} \]

In the example given here, the pressure of a
one-kilogram force spread over a 6-cm diameter
circle on your hand is 1 kg ÷ 28.3 cm² or 0.035
kilograms per square centimeter (abbreviated to
kg/cm²). The pressure from the kilogram with
the 1-cm bottom would be 1 kg ÷ 0.785 cm² or
1.127 kg/cm², while the pressure due to the
stiletto-bottomed mass would be 112.7 kg/cm².

(For those of you who have difficulty
extracting yourselves from the US culture, the
unit of pressure there is, as you may well know,
the pound per square inch, or "psi", often called
simply "pounds pressure". If you are ever
unfortunate enough to have to deal with a
pressure in this unit it can be converted to the
rest-of-the world unit by dividing by 2.93, or
about 3. Thus a pressure of 30 psi, a common
pressure for automobile tires, is actually a
pressure of about 10 kg/cm².)

A pressure of 112.7 kg/cm² is about 10 times
that of the air pressure in an automobile tire and
will come close to piercing your skin, depending
on how tough your skin actually is. The weight
would certainly pierce your skin if the mass was
sitting on a thumbtack pointing downward toward
your hand.

So it is not the weight of the object that will
damage your skin but the pressure. This is
generally true for all materials and is one of the
main reasons why the concept of pressure is
important. An architectural example arose in the
mid-sixties in North America when high stiletto
heels suddenly came into women's fashion. These
heels eventually became ridiculously small; in
some cases to about 5-mm diameter at their tips.
This resulted in a pressure from a 50 kg woman
resting most of her weight on just one of these
heels (a common poise for a woman suffering
from these torture devices) of about 150 kg/cm².
This pressure not only dented linoleum floors; it
even broke the edges of ceramic tiles.

In general, the ability of a material to resist
forces is related to the pressure that the material
can withstand in resisting these forces. This
pressure is related, in a very complicated way, to
the intermolecular forces within the material as
well as the type of deformation that the pressure
is tending to cause. For example, a dent is easier
to create in a soft material which can flow then it
is in a harder material that does not, even though
the harder material might break at a lower
pressure than the softer material. In technical
terms, the softer material is more "resilient". In
figurative sports terms, it "moves with the punch".
The result of this complication is that materials
have to be tested under various circumstances to
see what their strengths actually are for a
particular application. The following figures are
representative results for straight compression.
(See Hecht p. 349 Table 10.1.)

<table>
<thead>
<tr>
<th>Material</th>
<th>Compressive Strength (kg/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>350</td>
</tr>
<tr>
<td>Wood (soft)</td>
<td>500</td>
</tr>
<tr>
<td>Bone</td>
<td>1500</td>
</tr>
</tbody>
</table>

These figures may be at first surprising,
particularly that soft wood is stronger than
concrete and that bone is three times as strong as
wood. This comes from the ability of wood to
absorb pressure by deforming before breaking,
while concrete is brittle and breaks. Thus wood will dent but not break while concrete will not dent but will break instead. The strength of bone is a result of its marvelous complexity (it is the ultimate "composite material") that gives a combination of strength and ability to deform without breaking.

While pressure is easy to visualize as a squeeze, i.e. a compression, it can also be a pull as the various molecular bonds are stretched. Such an action can be thought of as a negative pressure. However, while it is still usually measured in kg/cm$^2$ it is normally referred to as a "tensile stress" rather than a "pressure". (A squeezing pressure is likewise often referred to as a "compressive stress".) In this case there is very little complication in the nature of the deformation of the material. The material just stretches until its molecular bonds break. Materials designed for this sort of resistance to movement tend to be long and fibrous. Some representative values for tensile strength of material are given in the table below. (For a more extensive list see Hecht p. 349 Table 10.1).

<table>
<thead>
<tr>
<th>Material</th>
<th>Tensile Strength (kg/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muscle</td>
<td>1</td>
</tr>
<tr>
<td>Bladder wall</td>
<td>2</td>
</tr>
<tr>
<td>Skin</td>
<td>100</td>
</tr>
<tr>
<td>Tendon</td>
<td>800</td>
</tr>
<tr>
<td>Hair</td>
<td>1900</td>
</tr>
<tr>
<td>Spider web</td>
<td>2400</td>
</tr>
<tr>
<td>Concrete</td>
<td>40</td>
</tr>
<tr>
<td>Wood (soft)</td>
<td>1000</td>
</tr>
<tr>
<td>Bone</td>
<td>1100</td>
</tr>
<tr>
<td>Ordinary steel</td>
<td>4000</td>
</tr>
<tr>
<td>Piano wire</td>
<td>3100</td>
</tr>
</tbody>
</table>

Again some of these numbers may be surprising, in particular the extreme strength of spider web (it is over half as strong as steel). Also the tensile strength of concrete might seem low until you realize that concrete is used to support weights that press against it rather than hang from it. In this respect, it may be surprising that bone material is so strong under both compression and tension but then bone has evolved to take a great variety of forces without breaking, some that try to stretch it while others try to crush it. A final number of great importance in the life sciences, particularly in sports medicine, is the low tensile strength of muscle. This comes about not by a neglect of nature but rather by the fact that nature has found no other fiber that can be activated by an electrical signal to contract on command. (Note that it has found a fiber, the material of the tendon, which does have a much higher strength but this is an inert material.) The low tensile strength of muscle fiber requires that a great deal of muscle mass is required for the sort of forces that are exerted in sports and until this muscle mass is achieved by gradual exercise it is very easy to create a muscle tear.

All of these stresses come about because of the requirement that the body create forces to resist forces that are applied to it from external agents. In other word, to balance these outside forces. I will therefore return to a more detailed consideration of this requirement of the balancing of forces.

**The Balancing of Forces**

The requirement that any force on a stationary body must be balanced by another force is one of the most important principles in physics. Yet, despite its importance, in the history of physics it was also one of the most difficult principles to discover, so don't feel inadequate if it is not at first obvious to you. To help, I will try to make it as believable as possible.

When you are yourself creating the upward force to resist the downward weight of an object the idea seems perhaps obvious. You are creating that force and you know it. It may not be so obvious when the force is being created by an insensate object such as a spring scale used for weighing amounts of material in the kitchen. However, if you look carefully at the reaction of such a scale when you put an object on it you will see that the pan of the scale moves down until the spring has built up sufficient upward force to stop the mass from falling any further. You can experience this building up of an upward force by pushing down on the scale yourself. The more you push the scale downward, the more it resists your push by pushing upward. In fact, at any instant the dial on the scale will read the force with which you are pushing down on it.

It is probably not at all obvious that any device that keeps an object from falling creates an upward force that must balance the downward weight of the object. Thus even if a book is resting on a table, that table must be creating an upward force exactly equal to the weight of the book.
It is here that, for most people, physics has gone too far. Tables are inanimate things that do not create forces all on their own! Yet that is exactly what physics tells you is happening.

Since all objects have weight all objects on Earth must be balanced by forces if they are not to move, and the vast majority of them do not move (they are, after all, called "inanimate").

It is natural then to consider a force as the weight that it would balance. Consider a spring scale, as used for weighing the fish you just caught. It registers the weight by a pointer that moves against a scale. This scale is calibrated so that when the device is used to suspend a fish from its hook the pointer is brought to the point on its scale that corresponds to the weight of the fish.

A very important concept here is that when a spring scale registers, say, 9.4 kg it is being pulled on from both ends by 9.4 kg. One way to think about this is to imagine the spring scale in the diagram turned upside down. It would still register 9.4 kg if a 9.4 kg fish was hung on it.

Of course, such a scale does not have to be used just for weighing fish. It can be used for measuring any force. For instance, if you connect a spring scale to a wall and pull on the scale it will read the force with which you are pulling against the wall.

Likewise, you can use it to measure the tension in a rope being used by two teams playing tug-of-war.

You could even use a set of these scales in a complicated tug-of-war between three teams, where one of the teams is much bigger than the others and so competes against the two smaller teams who help each other. The arrangement in the diagram below would allow you to tell exactly how the force is shared between the two smaller teams.

Three tug-of-war teams do not have to line up to play the game. They could arrange themselves in a triangular formation as shown in the diagram below, where now there are two bigger teams and one small team.

Here you would find that the one small team actually balances the two larger teams. The explanation for this is in the fact that force is a vector. Again, this is because force has both magnitude and direction. The force vectors involved in this bizarre tug-of-war are shown.
This method of having a small force produce a very large reaction force has been known from antiquity. A modern version of it can sometimes be seen in off-road expeditions where a vehicle has become stuck in rough terrain. (It does not appear as often as in the past because of electric winches and/or cellular telephones to call for emergency assistance.) The trick is to attach a steel cable to a draw-bar point on the vehicle and stretch it to a distant very large tree. The other end of the cable is then taken around the tree and, drawn tight, is wound around the tree several times. A helper then pulls on this cable so that it doesn't slip on the tree.

You then go to the middle of this cable and pull it sideways.

As an example, suppose the cable is 50 meters long and you pull it sideways with a force of 50 kg that actually pulls the rope a distance of 2 meters sideways. The force vector diagram for this is shown below.

Here the transverse force you have applied creates forces in the cable that balance your force when they are working in the directions shown. The components of these forces that are in the direction from the vehicle to the tree actually balance each other. It is the perpendicular components in the direction opposite to your force that work together to balance your force. Since the forces are symmetric they are equal in magnitude. Letting this magnitude be $T$, the components that balance your force are each

$$T_y = T \sin 45.57^\circ$$

Since there are two of them, the force being exerted by the cable on your hands is

$$2T \sin 45.57^\circ = 50 \text{ kg}$$

Solving this equations for $T$ gives

$$T = \frac{50}{2 \times \sin 45.57^\circ} = 313 \text{ kg}$$

Thus your 50 kg force has created a 313 kg reaction force in the cable. This is over a quarter of a Tonne, which is usually sufficient to move the vehicle. If it is not then you could ask a helper to add to your force so as to get over half a Tonne.

It is interesting to calculate how far your force would have moved the vehicle. The distance of the vehicle before you pulled on the cable was 50 meters. The distance is now

$$2 \times 25 \times \cos 4.59^\circ = 49.84 \text{ m}$$

So you have ended up only moving the vehicle by 16 cm. However that is usually enough to backfill stones against the tires to prevent the vehicle from rolling backwards. Your then release your pull and your helper takes up the slack in the cable by pulling on the end of the cable wrapped around the tree. The cable is then again pulled sideways at its center and the exercise repeated until the vehicle has been pulled out of difficulty.

There are at least three distinct points of view from which to consider this operation. First, consider it from the point of view of the vehicle (assuming vehicles have points of view, and from the behaviour of SUV’s it certainly seems that
they have. The forces on it are the force of the cable, being resisted, unsuccessfully you hope, by the force of the mud.

Thus the rope acts as a transmitter of the force. In the case of the vehicle being pulled out of the mud, the cable acts as a transmitter of the pulling force of the tree roots to the vehicle.

This action of a rope is one of the most important examples in mechanics. It has been used from antiquity to transmit forces over distances. It is also extremely important in the human body where the force of muscles is transmitted to bones by the tension in the tendons.

Compression can also be used to transmit a force over a distance. Examples of this are the use of a rod to push out a blockage in a sewer pipe or the hose of your household vacuum cleaner. Another example is the use of a poker to push a burning log in a fireplace.

Here the free-body diagram of the transmitter has inward pointing forces on its ends.

However, the forces that can be transmitted in this fashion are usually much less than those that can be transmitted by tension. This is because, while tension keeps the transmitter straight, compression tends to cause it to buckle. Once buckled, the device can support very little force, a flexible object like a rope being able to support virtually no compression whatsoever. (This is the basis of the cruel statement I first heard as an engineering student to express the contempt we had for what were then called "commerce" students but who are now more euphemistically referred to as "management"
students: "They are so dumb they don't know that you can't push on a rope".

**The Balancing of Forces on a Pulley**

One of the most important uses of tension as a force transmitter is in the pulley. While this device is of no direct interest in the life sciences, since the pulley is not found in nature but, like the wheel, is completely a man-made artifact, it should be of interest because of its importance in the history of human development. (Even medical school admission committees look with favour on any indication that a candidate is not a complete medical nerd.) Besides it is a very clear example of the use of free-body diagrams to understand what is going on in a system, and such diagrams are very important in understanding the internal forces within the human body.

The simplest and earliest application of the pulley was as a simple lifting device. It seems to have been invented shortly after the wheel, i.e. in prehistoric times. Many historians claim it to be the first true machine invented by man. A schematic of it is shown below.

![Schematic of a pulley](image)

This device used the ability of a rope to transmit a tension force, in this case over and around the pulley. In this way the upward force required to lift a weight was converted into a downward force. The importance of this lay in the easing of the forces involved in such lifting. When you lift an object in the ordinary fashion, the weight of the object being lifted adds to your weight on your knees. Thus a person with an upper body weight of 70 kg lifting a 50 kg weight will experience 120 kg of force on his or her knees.

This can cause considerable strain on the knees and, as will be considered later, on the major back muscles.

On the other hand, lifting the object by pulling downward on a rope lightens the load on the knees, in the above example to 20 kg. This can actually be beneficial to your knees and back. Of course, this method does not allow you to lift more than your weight. But then in general you should not be lifting more than your weight, particularly if you are to repeat this often in a day's work.

I said that the pulley is a good example for the use of free-body diagrams. For the single pulley the use is almost trivial, as in the diagram below where the body under consideration is the pulley itself.

![Diagram of forces on a pulley](image)

This diagram shows that there are two downward forces on the pulley, each equal to the tension in the rope. This tension is, of course, the downward pull you have applied which is, in turn, just the weight of the body. In the diagram this is indicated as W. These two applications of the weight W are balanced by the one upward force coming from the rope suspending the pulley from the ceiling by its axle. The tension in this rope is then twice the weight that is being lifted, an important point in designing the beam supporting the pulley.

Very shortly after the invention of the simple pulley, combinations of pulleys were used to enhance the lifting effect. Certainly these devices were well known to the ancient Greeks; the theory of pulley systems having been thoroughly
worked out by Archimedes. An example is the two-pulley system shown below.

Here the free-body diagrams for the pulleys are still relatively simple. The diagram for the lower pulley, which is lifting the weight, is shown below.

It shows that now the tension in the rope (i.e. the force with which you have to pull to balance the weight), is only half the weight. This is because there are two tensions acting on the pulley to balance the weight.

The diagram for the upper pulley is shown below.

This shows that the rope supporting this pulley, as in the case of the single pulley, must have a tension that is twice that of your pull. However, your pull is now only half the weight so this support cable will only have to withstand the weight itself.

Finally consider the support beam.

It shows that the roof support must now provide three times the force you have applied. This comes about again because of the necessity of balancing both your force and weight being lifted.

The free-body diagram technique can be applied in any combinations of pulleys. The final example that I will present is shown below for what is called a "three-pulley block" system. Normally such combinations of pulleys have all three pulleys of the same diameter and on the same axle but they are drawn here of different
diameters and on different axis to make the action clear.

The free-body diagram for the lifting pulley - block now shows that the force you have to apply, i.e. the tension in the rope, is only one-sixth of the weight. In other words, a 50 kg force will lift a 300 kg weight.

The diagram for the upper pulley block shows that the roof support must now supply seven times your force, or again, your force plus the weight of the object.

The Balancing of Pressures
As I have already said, in the real world the balancing of forces always come down to the balancing of pressures. This is because real objects have finite size and the force is spread over the area of contact. Since they share this area of contact, the two bodies will have the same compressive pressure.

However, there is another whole class of interactions involving liquids and gases where the balancing of pressure is more subtle. Again I will try to gradually introduce the subtleties of this
sort of interaction in the hope they can then be made more understandable.

First consider the application of a force to a piston in a chamber containing liquid.

The reaction pressure of the liquid due to this force is easy to calculate. This pressure supplies the reaction force required to balance the force applied to the piston. The force supplied by the pressure is just the pressure times the area of the piston. The pressure is therefore the force applied divided by the area.

\[ F = pA \]

\[ p = \frac{F}{A} \]

For example, if that force was 10 kg and the piston had a diameter of 1 cm, i.e. an area of 0.78 cm², the pressure would be 12.7 kg/cm². This is just the same as the pressure of a solid object in contact with another solid object.

Now consider the first subtlety for liquids. Consider the volume of liquid just below the piston and draw a free-body diagram of the forces on that volume.

The piston applies a downward force on the liquid that, for the liquid to be stationary, must be resisted by an upward force on the liquid. This must be provided by the liquid just beneath the section of liquid shown in the diagram. To balance, this pressure must be the same as the pressure downward from the piston. Therefore, just as the tension force is transmitted along a rope, the pressure is transmitted along the tube of liquid.

However, now consider the very subtle point concerning what happens if the tube expands to a larger diameter. (This point is so subtle that it was not fully understood until it was studied by the scientist Pascal in the 17th century in Europe.) As a specific example, suppose the tube expands from a diameter of 1 cm to a diameter of 10 cm. Taking the same force of 10 kg as in the previous case, resulting in a pressure of 12.7 kg/cm², we get the situation shown below.

The important question is; what is the pressure of the liquid in this larger tube?

In the seemingly similar case of a rigid object of this shape the calculation is simple. The upward force must balance the downward and so the upward pressure is reduced because of the larger area over which it acts.
For the example shown in the diagram, if the upper surface had a diameter of 1 cm at a pressure of 12.7 kg/cm² and the bottom had a diameter of 10 cm the pressure on the bottom of the object would be only 0.127 kg/cm².

However, the case for the liquid is not so simple. Unlike solids, liquids can flow. If put under pressure they will flow to press against any surface that restrains them, such as the walls of their containers. It is therefore the pressure that is transmitted by a liquid, not the force,

Thus while the effect is subtle the result is very simple. The pressure on the bottom of the volume of liquid under consideration here is the same as the pressure on the top!

This has enormous ramifications in liquid systems, ranging from machinery for supplying very large forces to very heavy objects (where the subject is called "hydraulics") to the heart pumping blood through your arteries to supply your brain. Because the machinery aspect is simpler I will consider that first.

Returning to the above example, the pressure in the liquid produced as a reaction to the applied force of 10 kg on the 1-cm diameter piston would also act on a 10 cm diameter piston in the bottom tube.

The result would be a force in proportion to the area of the piston. In other words, the force on this piston would be 1000 kg, or one Tonne! Since the liquid presses against any surface, no matter how it is oriented, the geometry can be rearranged so that the larger piston also faces downward. The force of the liquid is then upward, allowing it to support a weight of 1 Tonne. Thus a 10-kg force could raise a mass of 1 Tonne.

This ability of pressure to produce very large forces is very important in the construction of automobile tires, where a relatively modest pressure of 10 kg/cm² in four tires can support over a Tonne of machinery hurtling along a highway at 160 km/hr (at least in Germany where such speeds are legal). It is even important in the construction of bicycle tires, where a pressure of about 20 kg/cm² can support the weight of a 100 kg rider. In the life sciences it is extremely important because of the forces produced on the heart walls by the very modest blood pressure (about 0.15 kg/cm²) required to push blood through the arteries.

**Pressure Due to Depth in a Liquid**

So far I have considered the pressure in a liquid from a force applied to a piston. However, there is another very important source of pressure in a liquid; that due to its own weight. This pressure is well known to deep-sea divers.

That this pressure is due to the weight of the water can be seen from the free-body diagram of the water above you. For example, consider the free-body diagram of a column of water one centimeter square and extending from your ear to the ocean surface. Suppose also that your ear is 30 meters below the surface.
The weight of this column of water, assuming the density to be same as ordinary water, which is good enough for this type of calculation, would be 2.36 kg. The force on the bottom of this column to support this weight would therefore also have to be 2.36 kg. The area of the bottom of this column is \( \pi/4 \) cm\(^2\). The pressure at the bottom is therefore 3 kg/cm\(^2\).

If the water is pressing against your ear, then your ear must provide a pressure of 3 kg/cm\(^2\) to keep it out. This is the source of the pain if you dive suddenly into deep water. You can only safely take deep dives if you allow the pressure on your ear drums to equalize by allowing high-pressure air to seep through the canal leading to the interior behind the eardrum so that the eardrum itself is not required to provide all the necessary resistance to the water pressure.

From these numbers you can see that 3 kg/cm\(^2\) at 30 m of depth corresponds to 0.1 kg/cm\(^2\) per meter of depth. If you convert the pressure to kg/m\(^2\), by simply multiplying by 10000, you get that the pressure is 1000 kg/m\(^2\) per meter, or 1000 kg/m\(^3\). This is, of course, just the density of water. Thus pressure rises in proportion to the depth, the proportionality constant being just the water density. In equation form

\[ p_{\text{kg/m}^2} = \rho h \]

where \( \rho \) is the internationally approved symbol for density and \( h \) is the depth of water.

This variation of pressure with height of liquid that is supported is very important in many mammals, particularly those that have their heads far above their feet. In humans it results in a blood pressure that is significantly higher at the feet than at the brain, where the blood is most needed. This is why you often feel momentarily dizzy when you get up suddenly, before the heart has livened up to provide the increased pressure needed to get blood to the brain when you are standing. It is also why putting up your feet as often as you can is very beneficial to blood circulation in your legs. (Not as much pressure is then needed to force the blood out of your feet and back to your heart.) As an example with numbers, a person whose brain is 60 cm above his heart will have a blood pressure at his brain that is 0.06 kg/cm\(^2\) less than at his heart.

This problem is very severe for the giraffe, where the head can be several meters above the heart and a special blood circulation system is needed to get blood up to its brain. This is also probably one of reasons why the brontosaurus dinosaurs had such small brains. It was just too much effort to pump the blood to the brain when they reached to eat high leaves.

**Buoyancy**

A very important consequence of the variation of pressure with depth is the buoyancy of objects in water. To make the arithmetic simple, consider a rectangular block that is floating.

The water underneath this object must be supplying the force needed to prevent the object from sinking any farther. Since water can only supply force as a pressure it is then, strictly speaking, the pressure of the water that is holding up the block.

For a rectangular block floating with its edges vertical, the pressure on the sides produces only horizontal forces that have no components that can resist the weight of the block. The weight must therefore be resisted by the pressure on the bottom of the block.

The force produced by this pressure is the pressure times the bottom area of the block. This pressure (in kg/m\(^2\)) is the density of water times the height of the surface above the bottom. The force due to this pressure is therefore the density of the water times the depth of the bottom of the block times the area of the bottom of the block. This is just the density times the volume of water
that the block displaces. Thus the block is buoyed up by a force equal to the weight of the water it displaces.

\[
\text{FORCE} = \text{WEIGHT OF WATER DISPLACED} = \text{WEIGHT OF OBJECT}
\]

This is the famous theorem of Archimedes which, according to legend, he discovered while taking a bath and which caused him to leap out of his bath and run naked down the street shouting Eureka!

The reverse way of looking at this is that a floating object displaces an amount of water equal to its own weight. This is often demonstrated in high-school science courses by placing a floating object in a container for which any rise in the water level results in water spilling out into another container. The water collected in the other container can then be compared with the weight of the object. For example, floating a 200 gm object would result in 200 ml of water being spilled into the collection container.

What is not so obvious to most people is that even an object that sinks will have a buoyant force acting on it. Consider an object with the same rectangular shape as before but too heavy to float. Since the object sinks, the water pressure on its bottom does not supply sufficient force to hold the object in place. Some other force is needed, such as that from a rope tied to the object.

But what is the size of the extra force that is needed? Most people think that it is the same as if the water was not there. However, you can easily prove this to be wrong. An easy demonstration of this is to tie a rope to a concrete block, which weighs about 10 kg, and lower it into a garbage can almost filled with water. (The hardest part of this demonstration is getting rid of the water afterwards, a problem that has often caused problems between physics teachers and school custodians.)

You will find that lowering the block into the water makes it seem surprisingly lighter. In fact, a typical 10 kg concrete block will weigh only about 5.5 kg under water.

This effect is explained by the difference in pressure at the top of the block compared to the bottom. This difference is just due to the difference in the height of the top and the bottom of the block. This difference is, of course, just the height of the block and the pressure difference is this height times the density of the water. Multiplying this pressure difference by the area over which the pressures acts gives the density of the water times the volume of the block. Thus the block is still buoyed up by the
weight of water the block displaces. The only difference compared to the floating block is that now the block displaces an amount of water equal to its volume, not to its weight. The block actually sinks because, due to its density being higher than that of water, this amount of water does not weigh as much as the block itself. In other words, the buoyant force is insufficient to support the block and so an extra support force must be supplied to make up the difference.

**Pressure Due to The Atmosphere**

The concept of a pressure change due to differences in elevation is very important in atmospheric phenomena. In modern society it is well known that changes in altitude result in changes in atmospheric pressure, the higher you go the lower the pressure. It is often experienced as a feeling in your ears during take-off and landing in modern air travel. (It is not nearly as painful as it used to be before they had pressurized cabins to counteract the effect.) It is also well known in the difficulty of simply boiling an egg in places like Denver, or as high-altitude sickness during strenuous athletics in places like Mexico City. These are due to the thinness of the atmosphere resulting from the reduced pressures at these elevations. This lowers the temperature at which water boils and reduces the transfer of oxygen from the lungs to the blood.

What is not so well know is exactly how much the pressure changes with height and what causes it. From the consideration of the pressure change with depth in water it must be just the result of the weight of air between the two elevations.

Most people do not even realize that air has weight. However, at sea level it is about 1.3 kg/m³. Perhaps an easy way for you to visualize this is to consider having an extremely low temperature refrigerator that can liquefy air. (They do exist and are widely used in medicine to produce liquid oxygen and nitrogen.) If you then filled a balloon with one cubic meter of air (to a diameter of about 1.25 m) and fed all this air into the refrigerator you would end up with 1.3 kg of liquid.

Thus the pressure drop from sea-level to 1 meter above sea-level is 1.3 kg/m², the pressure drop at 2 meters would be 2.6 kg/m², etc. (To obtain these pressures in the more usual gm/cm² just divide by 10. The pressures then become 0.13 gm/cm² and 0.26 gm/cm² respectively.)

However, unlike water, as you rise in air it gets thinner. It therefore weighs less. The pressure drop per meter is therefore less. In other words, unlike water, where the pressure drops smoothly to exactly zero at the surface, the pressure of air fades away as you go into outer space. There is no sharp point at which it becomes zero. There is no "edge" to the atmosphere.

Yet in deep space there is no air and so its pressure must be zero. What then is the change in pressure from the Earth's surface to outer space. This must be just the pressure of air at the Earth's surface.

But what is this?

For a long time humans did not even realize there was such a pressure. This is because air pervades everything on the Earth's surface and therefore equals its pressure against all surfaces. For example, the pressure of the air on the outside of your eardrums is balanced by the pressure of the air on the insides of your eardrums and there is no net force on your eardrums.

It is only when the air on one side of a surface is removed, such as when you pump air out of a container, that the pressure of air can be seen. And it is enormous! This was first demonstrated in the Middle Ages in Europe when the air was pumped out of a chamber formed by two hemispheres of about 60 cm in diameter. The force of the atmospheric pressure on these hemispheres was so great that two very strong horses could not pull them apart. A more modern demonstration of such forces, which is often performed in high-school physics courses by removing the air from an ordinary container, is illustrated by a photograph in Hecht.

Again, careful physics measurements, in which all the air has been removed from a chamber and the resulting forces accurately measured, has shown normal atmospheric pressure at sea-level to be about 10 Tonnes per square meter! (That is about 1 kg/cm².)

As a concluding thought, this measurement of the atmospheric pressure at sea-level actually determined the total mass of the Earth's atmosphere. Since the pressure on the surface is due to the weight of all the atmosphere directly above that surface, then the total weight is the atmospheric pressure times the surface area of the Earth. The surface area was known in the 16th century to be about $5 \times 10^{14}$ m². The total mass
of the atmosphere is therefore about $5 \times 10^{15}$ Tonnes, or about $5 \times 10^{18}$ kg.

In many operations on Earth, such as determining the pressure underwater, it is the deviation of a pressure from normal atmospheric pressure that is important. This is obtained by simply subtracting 1 kg/cm$^2$ from the actual pressure. For example, when you pump air into an automobile tire the pressure is actually 1 kg/cm$^2$ when the tire is flat. What you then want to know is how much the pressure has actually increased by pumping in the air. Thus when the pressure is actually 4 kg/cm$^2$, the pressure gauge on your filling hose will read 3 kg/cm$^2$. To distinguish this pressure from the "Absolute Pressure", i.e. 4 kg/cm$^2$, this pressure is often referred to as the "Gauge Pressure". (See Hecht.)

**The Newton and The Pascal**

By now I can almost feel the wrath of hordes of physics teachers who are loudly protesting that force and pressure are not measured in kg and kg/cm$^2$. My response to this is that 99% of the human race, who even understand what force and pressure are all about, do measure them and talk about them this way. (Except, again, for those poor souls who have to contend with US units.) It is only the physicists, and a few chemists and other physical scientists, who do not use these units. Certainly most of the engineers of the world use them.

Why then do the physicists make such a big fuss about them?

The problem arises for objects where the force is the weight of a mass. This results in a double meaning for the word "kilogram", often in the same sentence. Thus the weight of a one kilogram object is one kilogram, a statement that seems utterly redundant until one realizes the word kilogram in this sentence has referred to both the amount of matter (i.e. its mass) and the force on it.

To remove this confusion, which can be very confounding when it comes to the motion of masses (see lecture 21) physicists use, exclusively, Newtons (abbreviated N) to specify forces. The reason for this will be a subject for Lecture 21 but for now you only need to know that it is a force of about 100 grams, or one-tenth of a kilogram. More accurately, it is the weight of 102 grams, so that one kilogram weighs 9.8 Newtons.

You may have already suspected, even if you haven't come across it before, that this is the acceleration of gravity on the surface of the Earth. The number 9.8 is so important in mechanics that, by International agreement, it is designated by the symbol $g$.

Thus a physicist will say that a 1 kg mass weighs 9.8 N, thereby implying that the force of gravity on the mass is 9.8 N.

Similarly for the units of pressure. Here the International unit for area is the square meter so the International unit for pressure is the Newton per square meter. This mouthful has been redefined as the Pascal (abbreviated to Pa), in honour of Pascal's studies of pressure. A conversion, good enough for elementary physics courses and other university work is just to multiply kg/cm$^2$ by 100,000 to get Pascals. Thus normal atmosphere is about 100 kPa.

In these SI units, the equation for the pressure versus height of liquid becomes

$$p = \rho gh$$

and this is the way you will deal with it in the rest of the course. In fact, to appease the souls of deceased physics teachers I promise not to use force as a kilogram or pressure in kg/cm$^2$ ever again (well almost never again) in this course.

You might ask why did I use kg and kg/cm$^2$ in the first place? I did so because I have found that the difficult concepts of force and pressure become even more bewildering to beginning students if they are introduced using the totally unfamiliar units of Newtons and Pascals.

**Relevant material in Hecht**

Section 4.2, solved problems 4.1 and 4.2, Section 4.9 (Introduction to, but not including “Parallel Force Systems”, solved problems 4.11 and 4.12, Section 9.4, with solved problems 9.3 and 9.5, Section 9.5 with solved problem 9.6 and Section 9.6 with solved problems 9.7 and 9.8

**Solved Example Problems**

1. The Figure shows two blocks, block 1 and block 2, held in equilibrium by massless, unstretchable strings. The mass of block 1 is 18.2 kg, the mass of block 2 is 3.8 kg and the angle $\theta$ is 25°. What are the tensions in the strings?
The diagram, with numbers, is shown below.

The free body diagram of the point where the three strings meet is

The tension $T_2$ is just that resisting the weight of the 3.8 kg mass, i.e. 37.24 N.

The other forces can now be obtained by taking the vector sum to be zero.

2. A dentist's chair of mass 233 kg is supported by a hydraulic lift having a large piston of cross-sectional area 1410 square centimeters. The dentist has a foot pedal attached to a small piston of cross-sectional area 74 square centimeters. What force must be applied to the small piston to raise the chair?
3. A cord runs around two pulleys as shown below. 1.67 kg bucket hangs from one pulley. If the mass of the pulleys and the friction of the system are negligible, by how much must you pull on the rope if you are to lift the bucket?

\[ F = ? \]

\[ m = 1.67 \text{ kg} \]

Free body diagram of pulley B

\[ \text{Balance of forces} \]

\[ W = 2F \]

\[ F = \frac{W}{2} = 8.18 \text{ N} \]