1. **2 pts**

We can determine that a black hole is present by observing the effects of its gravitational force on surrounding objects. For example, observing the orbital motion of the stars in the region can allow us to infer the existence of the black hole. We can also determine its mass by observing the period of this orbital motion and applying Kepler’s third law. Another indirect evidence would be observation of X-ray radiation from a disk of material that has been accreted by the black hole. One could also observe effects of gravitational lensing on the images of stars/galaxies along the line of sight to the black hole.

2. **2 pts**

To get the maximal radius, we can set the escape velocity equal to the speed of light i.e. when the gravitational force does not allow light to escape the surface of the black hole. Escape velocity is given by,

\[
v_{esc} = \sqrt{\frac{2GM}{R}}
\]

(1)

Setting \( v_{esc} = c \) and solving for \( R \), we get,

\[
R = \frac{2GM}{c^2} = 2.95 \times 10^9 m,
\]

(2)

where \( c = 3 \times 10^8 \text{ m/s} \), \( M = 2 \times 10^{36} \text{ kg} \) and \( G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg s}^2) \)

3. **2 pts**

A Level 1 absorption line can be observed if electrons in the ground state jump to higher energy states by absorbing photons of the appropriate energy (wavelength) from an emission source. Here, the emission source is the star with the electrons in the hydrogen envelope of the star being responsible for the absorption lines. Another possible source of the absorption lines could be an intervening cold hydrogen cloud between the star and the observer. The width of these absorption lines would depend on broadening caused by Doppler shifts due to the thermal motions of atoms. The lines could also be broadened due to changes in the energy levels of electrons due to high densities in the hydrogen envelope/gas cloud.

4. **4 pts**

The oscillation in the frequency of the emission peaks can be explained by the rotating disk of material that is present around the supermassive black hole in a quasar. This is a result of observed Doppler shifting due to parts of the disk moving towards us while other parts moving away from us. The observed emission frequency of the regions moving towards us will be higher i.e. blue-shifted and for the regions moving away from us, the observed frequency would be lower i.e. red-shifted, thereby explaining the oscillation.

Knowing that the oscillations in frequency are of the order of 1%, we can calculate the velocity of material in the disk. Approximating the oscillations in the emission frequency as the oscillations in wavelength,

\[
\frac{\Delta \lambda}{\lambda} \sim \frac{v}{c}
\]

(3)

\[
v = 0.01 \ c = 3 \times 10^6 \text{ m/s}
\]

(4)

5. Included above.
6. 7 pts

(a) Each element shows emission and absorption lines of unique wavelengths, corresponding to the structure of the energy levels within the atom. Observing the wavelengths of specific emission and absorption features can allow us to ascertain the composition of an object.

(b) If the object has a spectrum resembling a blackbody, then the temperature can be determined by knowing the peak wavelength and applying Wein’s displacement law. Additionally, observing spectral lines of ions vs. observing those of molecules can help determine the temperature if the object does not have a continuous blackbody spectrum. This is because ionization of an atom requires high temperature whereas existence of molecules requires low temperatures in order to avoid them breaking up into constituent atoms.

(c) Only particular emission lines can be observed from a low density gas cloud. Something more substantial (say a star or planet) would instead have a continuous spectrum.

(d) An upper atmosphere will result in absorption lines in the spectrum emitted by the object. With a high enough temperature, it can also result in emission lines like the UV emission lines observed in the spectrum of Mars.

(e) The object would have a spectrum similar to that of the star (a continuous blackbody spectrum) but greater intensity of blue light as blue light is scattered most effectively.

(f) The speed of the object can be determined by observing the change in the frequency of an observed spectral line due to Doppler shifting (Eq. 3).

(g) The spectral lines from the part of the rotating object moving towards us will be blue-shifted and that moving away from us will be red-shifted. The extent of the rotational broadening of spectral lines can allow measurement of the velocity (Eq. 3).

7. 2 pts

The frequency of the photon is given by,

\[ \nu = \frac{c}{\lambda} \]

\[ = \frac{3 \times 10^8 \text{ m/s}}{120 \times 10^{-9} \text{ m}} \]

\[ = 2.5 \times 10^{15} \text{ Hz} \]

The energy of the photon is,

\[ E = h \nu \]

\[ = 6.626 \times 10^{-34} \text{ J s} \times \nu \]

\[ = 1.66 \times 10^{-18} \text{ J} \]

8. 3 pts

(a) The energy of the photon is given by,

\[ E = \frac{hc}{\lambda} = 3.31 \times 10^{-19} \text{ J} \]  (5)
(b) The photons emitted, \( N \), should have a total energy of 100 J/s.

\[
N = \frac{100 \text{ J/s}}{3.31 \times 10^{-19} \text{ J/photon}} = 3 \times 10^{20} \text{ photons/second} \quad (6)
\]

(c) Owing to the large number of photons emitted per second from ordinary light sources, it is not possible to perceive or detect the individual particles.

9. 4 pts

(a) Using Wein’s displacement law,

\[
\lambda = \frac{2.9 \times 10^{-3} \text{ m K}}{3000 \text{ K}} = 966 \text{ nm} \quad (7)
\]

(b) Since the peak wavelength is greater than that of solar emission, we expect the light to be redder. In the context of films used for indoor photography, this difference in peak wavelengths between the sun and a light bulb can explain why indoor photography films have enhanced sensitivity to blue light. The enhanced sensitivity helps correct the image colors for the additional ‘redness’ induced by indoor lighting.

(c) The light from the 3000 K filament has a blackbody spectrum peaking at the infrared wavelengths with only a fraction of the energy being emitted in the visible part of the spectrum. The infrared radiation is absorbed by the glass encasing of the filament and makes it hot to touch.

(d) As explained in the previous part, most energy used to heat up the tungsten filament does not end up being emitted as visible radiation. On the other hand, since fluorescent bulbs have emission lines at specific visible wavelengths, they can produce more visible light for the same wattage.

(e) Since fluorescent bulbs produce more visible light for the same wattage, they can end up saving money in the long run. Although their initial cost is higher, factors like their increased lifespan will result in greater cost-efficiency.

10. 4 pts

(a) The angular separation \( \theta \) is given by,

\[
\theta = \frac{r}{D} = 8.23 \times 10^{-6} \text{ radian} \quad (8)
\]

where \( r \) is the separation between Sun and Jupiter \((778 \times 10^6 \text{ km})\), while \( D \) is 10 light years \((9.46 \times 10^{13} \text{ km})\). Converting radians to arcseconds, we get an angular separation of 1.69”.

(b) The angular separation between Sun and Earth at the given distance can be calculated using the same formula to be 0.33”.

(c) The angular resolution of Hubble space telescope is \( \sim 0.05” \) i.e. both these angular separations can be resolved. Thus the challenge of observing planets around other stars does not arise from us not being able to resolve the planet from the star but rather due to the light from the star being orders of magnitude brighter than the light the planet reflects i.e. not being able to see the planet in the ‘glare’ of the star.