PHYS 182 - Homework 4 Answers sheet

November 7, 2018

1

The observer at rest measures $\delta t_{\text{rest}}$ and the traveler measures $\delta t_{\text{traveler}}$. The traveler is circling the equator 10 times at a speed of $0.9c$. Therefore, the time he takes for his journey is

$$\delta t_{\text{rest}} = \frac{v}{d} = \frac{(20\pi \times 6378 \times 10^3) \times 10}{0.9 \times 3 \times 10^8} = 1.48s$$  \hspace{1cm} (1)$$

The distance $d$ traveled is equal to 10 times the perimeter of the equator and $v$ is its speed.

$$\delta t_{\text{traveler}} = \gamma \times \delta t_{\text{rest}}$$  \hspace{1cm} (2)$$

where $\gamma$ is the Lorentz factor.

$$\delta t_{\text{traveler}} = \sqrt{1 - \frac{v^2}{c^2}} \times 0.48 = 0.645s$$  \hspace{1cm} (3)$$

2

$$v_{\text{observed}} = v_{\text{source}} \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$  \hspace{1cm} (4)$$

The source frequency is given as $v_b$ for a blue laser. The typical frequency corresponding to a blue wavelength is within a range of 610 to 670 THz. Let $v_b = 640$ THz.

$$v_{\text{observed}} = v_b \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$  \hspace{1cm} (5)$$

$$v_{\text{observed}} = 640 \times 10^{12} \sqrt{\frac{1 + \frac{5}{3}}{1 - \frac{5}{3}}} = 1.68 \times 10^{15}Hz$$  \hspace{1cm} (6)$$

Furthermore, the wavelength is given by

$$\lambda = \frac{c}{v} = \frac{3 \times 10^8}{1.68 \times 10^5} = 1.79 \times 10^{-7}m = 179nm$$  \hspace{1cm} (7)$$

This frequency/wavelength corresponds to ultraviolet radiation according to the electromagnetic spectrum.
The observer sitting beside the garage sees the car shorter and contracted compared to its actual size. The driver on the other hand, who is in a moving frame, approaching the garage, sees the garage contracted, as is shown by the following calculation where the rest length of the garage is 8m.

\[
\text{Length in the moving frame} = \sqrt{1 - \frac{v^2}{c^2}} \times 8 = 5.33\text{m}
\]  

(8)

The observer at rest sees the car as being 6 meters long and the garage 8 meters long. Therefore, understanding that the car will fit in the garage. However, for the driver, the garage appears to be 5.33 m and the car he is in is 9 meters long. The driver does thus nor believe that the car will fit in the garage. They reach different conclusions due to the length contraction described in the theory of special relativity.

The expansion of the universe can be compared to the analogous model of raisin bread. In this model, even though the dough is expanding, the raisins themselves do not expand. Their relative distance to each other increases as space expands. The wooden stick does not expand, because atoms are held together by electromagnetic forces, and these forces in short range are much greater than the force causing expansion.

The equation given for the expansion of the universe is \(H^2 = \frac{8\pi G}{3} \rho\) with \(H\) being the expansion rate, \(G\) Newton’s gravitational constant and \(\rho\) the energy density. \(G\) is a constant, so this suggests that \(\frac{8\pi G}{3}\) is a constant. Therefore, \(H^2\) is proportional to \(\rho\). However, the universe expands at an accelerated rate. If we plot Hubble’s constant as a function of time, we realize that the further back in time, the larger \(H\) is. \(H\) is infinitely large as we go back in time, this implies that \(\rho\) was also infinitely large, or the volume is infinitely small.

Heisenberg’s uncertainty relation is given by:

\[
\Delta x \Delta p \geq \frac{\hbar}{4\pi}
\]  

(9)

where \(\Delta x\) is the uncertainty in position, \(\Delta p\) is the uncertainty in momentum and \(\hbar\) is the Planck’s constant. \(\Delta x = 10^{-10}\text{m}\)

\[
\Delta p = m \times \Delta v
\]  

(10)

Thus,

\[
\Delta p \geq \frac{\hbar}{4\pi \times \Delta x}
\]  

(11)

\[
\Delta v \geq \frac{\hbar}{4\pi \times \Delta x \times m}
\]  

(12)
\[ \Delta v \geq \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-10} \times 9.11 \times 10^{-31}} \]
\[ \Delta v \geq 5.58 \times 10^7 \text{m/s}^{-1} \] (13)

\[ \Delta v \geq 5.58 \times 10^7 \text{m/s}^{-1} \] (14)

9

The method to follow in this question:

\[ \Delta x \Delta p \geq \frac{\hbar}{4\pi} \] (15)

where \( \Delta x = 10^{-1} \text{m} \).

\[ KE = \frac{p^2}{2m} \] (16)

\[ E = m \times c^2 \] (17)

\[ \Delta E = KE + E \] (18)

\[ \Delta E \Delta t \geq \frac{\hbar}{4\pi} \] (19)

The time a virtual particle would spend in this box is:

\( \Delta t = \ldots \)

10

The Hydrogen atom only possesses one electron orbiting its nucleus that emits the energy it would require to stay in orbit as light, because it is accelerating. The accelerating charge produces electromagnetic radiation leading to a loss of energy as the electron orbits. Therefore, Hydrogen is considered unstable in classical physics, according to Bohr’s model of the atom. In quantum mechanics, however, the electrons can only be in quantized energy levels. In other words, they are only allowed to be in those specific levels unless they absorb enough energy to move from one level to another or completely break free.