1. **3 pts**

   To undergo pair production i.e. produce an electron, $e^-$ and a positron, $e^+$, a photon needs to have energy ($E$) corresponding to the total mass of the electron ($m_{e^-}$) and the positron ($m_{e^+}$).

   $$E = (m_{e^-} + m_{e^+}) c^2$$
   $$= 2 \times 9.11 \times 10^{-31} \text{ kg} \times (3.0 \times 10^8 \text{ m/s})^2$$
   $$= 1.64 \times 10^{-13} \text{ J}$$
   $$= 1.024 \text{ MeV}$$

2. **7 pts**

   If two protons collide to produce a Higgs particle then the energy of each proton should be equal to half the energy corresponding to the mass of the Higgs particle, $E_H$, which is,

   $$E_H = 100 \ m_p \ c^2$$
   $$= 1.51 \times 10^{-8} \text{ J}. $$

   Equating this energy with that of a proton moving with relativistic velocity, $v$,

   $$\frac{E_H}{2} = \frac{m_p c^2}{\sqrt{1 - v^2/c^2}}$$
   $$\sqrt{1 - v^2/c^2} = \frac{2 \ m_p c^2}{100 \ m_p c^2}$$
   $$\frac{v}{c} = \sqrt{1 - \left(\frac{1}{50}\right)^2}$$
   $$v = 0.99995c = 2.9977 \times 10^8 \text{ m/s}$$

3. **7 pts** The uncertainty principle for position and momentum is given by:

   $$\Delta x \Delta p \geq \frac{h}{4\pi},$$

   where $h$ is the Planck’s constant. The uncertainty in the position, $\Delta x$ of the proton will be equal to the size of the universe, $R$, which is related to the temperature, $T$, by:

   $$R = \left(\frac{T_0}{T}\right) R_0$$
   $$= \frac{3 \text{ K} \times 10^{10} \text{ Mpc}}{T} = \frac{9.3 \times 10^{32} \text{ K m}}{T}$$

   where $T_0$ and $R_0$ are the temperature and size of the universe at present time.

   The maximum uncertainty for one proton is,

   $$\Delta p = \Delta (mv) = m_p c$$
   $$= 5.0 \times 10^{-19} \text{ kg m/s}.$$ 

   Substituting the values of $\Delta x$ and $\Delta p$ in the uncertainty principle and solving for $T$,

   $$\frac{9.3 \times 10^{32} \text{ K m}}{T} \times 5.0 \times 10^{-19} \text{ kg m/s} \geq \frac{h}{4\pi}$$
   $$T \leq 8.8 \times 10^{48} \text{ K}$$
4. 4 pts
Since temperature of the universe scales as $t^{1/2}$, it will become hotter as we go back in time. At really high temperatures, say around $10^{12}$ K, a proton would break down into its constituent particles, the quarks, and form a quark-gluon plasma. This change happens at a high enough temperature such that the kinetic energy of the quarks becomes larger than the binding energy of the strong force holding the quarks together.

5. 5 pts
   a) The gravitational force between two electrons with masses $M_1$ and $M_2$ and distance $d$ between them is,

   \[ F_G = \frac{GM_1M_2}{d^2} = \frac{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \times (9.1 \times 10^{-31})^2 \text{ kg}^2}{(10^{-10})^2 \text{ m}^2} = 5.52 \times 10^{-51} \text{ N} \]

   where $M_1 = M_2 = 9.1 \times 10^{-31}$ kg and $d = 10^{-10}$ m.

   b) The electromagnetic force for electrons with charges $q_1$ and $q_2$ and distance $d$ between them is,

   \[ F_{EM} = \frac{kq_1q_2}{d^2} = \frac{9.0 \times 10^9 \text{ N m}^2 \text{ Coul}^{-2} \times (-1.6 \times 10^{-19})^2 \text{ Coul}^2}{(10^{-10})^2 \text{ m}^2} = 2.30 \times 10^{-8} \text{ N} \]

   where $q_1 = q_2 = -1.6 \times 10^{-19}$ Coulomb.

   c) The electromagnetic repulsion is $F_{EM}/F_G = 4.16 \times 10^{42}$ times stronger than the gravitational attraction between them.

6. 6 pts
   a) The gravitational force between Earth & Sun is given by,

   \[ F_G = \frac{GM_{\text{Earth}}M_{\text{Sun}}}{d^2} = \frac{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \times 6 \times 10^{24} \text{ kg} \times 2 \times 10^{30} \text{ kg}}{(1.5 \times 10^{11})^2 \text{ m}^2} = 3.56 \times 10^{22} \text{ N} \]

   b) Electromagnetic force between Earth & Sun for one excess electron each,

   \[ F_{EM} = \frac{kq_1q_2}{d^2} = \frac{9.0 \times 10^9 \text{ N m}^2 \text{ Coul}^{-2} \times (-1.6 \times 10^{-19})^2 \text{ Coul}^2}{(1.5 \times 10^{11})^2 \text{ m}^2} = 1.02 \times 10^{-50} \text{ N} \]

   where $q_1 = q_2 = -1.6 \times 10^{-19}$ Coulomb.

   c) Gravity is always attractive i.e. there are no negative masses. However, electromagnetic force can be attractive or repulsive. Since the universe is mostly electrically neutral i.e. most objects have an equal number of positive and negative charges, the sum total of their mutual electromagnetic interactions cancel out over large scales, which is why gravity dominates.
7. 4 pts
Using Wein’s displacement law with the given temperatures,

$$\lambda_{max} T = 0.00289 \text{m K}$$ \hspace{1cm} (1)

with temperatures of 5800 K and 4000 K, we get wavelengths of 498 and 722 nm. These correspond to green and red colors on the spectrum.

8. 3 pts
The total energy that can be derived by converting 0.7% of mass of an average human body (say, 70 kg) to energy using fusion:

$$E = 0.007 \times 70 \text{ kg} c^2 = 4.41 \times 10^{16} \text{J}$$ \hspace{1cm} (2)

where \(c\) is the speed of light in m/s. The time for which this energy can sustain us, assuming we use 100 J per second, will be \(4.41 \times 10^{14}\)s.

9. 7 pts
Doppler shifting in the solar spectrum allows observation of the radial motion of the solar surface. Such observations have shown that the solar surface pulsates i.e. expands or contracts on the timescales of minutes. Studying these acoustic oscillations is important to figure out the properties (such as the magnetic field, gravity and composition) of the medium that these waves have gone through, that is the interior of the sun.

Due to their weak interactions with matter, neutrinos are another probe of the solar interior, particularly the fusion reactions going on in the interior. These particles emitted during fusion in the solar interior can traverse the extent of the Sun very easily as compared to photons and be detected by neutrino detectors on Earth.

10. 4 pts
Nuclear fusion in the Sun would be more efficient if the strong nuclear force was ten times as strong. This is because greater attraction between nuclei would compete against the repulsive electromagnetic force, thereby requiring less energy for the nuclei to fuse. As a result, the internal temperature of the sun would be much higher.