Homework #6  (Week of Nov 5, 2018)

Problem #1

- Yes, it is possible that a civilization in another star system may see a neutron star as a pulsar, even if we observe no pulses of radiation from our observation point in our solar system.
- A neutron star is the compact corpse of a high-mass star left over after a supernova, while a pulsar is a neutron star from which an observer may observe rapid pulses of radiation as it rotates. The pulsations of the pulsar result from the neutron star spinning rapidly, following the conservation of angular momentum. Moreover, the collapse of the iron core bunches the magnetic lines running through the core tightly, and the intense magnetic field of the neutron star (pulsar) directs beams of radiation out aligned the magnetic poles. In other words, the pulsar beams radiation along its magnetic axis. It is possible that such a spinning neutron star is oriented such that its beams do not sweep by our location, in which case we would not observe a pulse of radiation, but that its orientation causes its beams to sweep across the civilization in another star system, who would then be able to observe a pulse of radiation. They would be able to recognize it as a neutron star because only an object as small and dense as a neutron star could spin at such extreme velocity without breaking apart.

Problem #2

X-ray bursts arise from the sudden ignition of helium fusion on the accreting neutron star in a close binary system. In theory, bright X-ray radiation should also be detectable from a close binary system containing a black hole instead of a neutron star. On one hand, it is true that no information (including X-ray radiation) can escape the event horizon of a black hole due to its gravitational pull, and black holes themselves emit no light. However, similarly to neutron stars, a black hole may be surrounded by a hot, X-ray emitting accretion disk. It is possible that the high-temperature gas in the accretion disk surrounding the black hole emit X-rays bursts somehow similar to X-ray bursts.

Problem #3

The radius of the event horizon of the black hole resulting from the merger of a black hole binary system would be smaller than the sum of the event horizon radii of the two original black holes.
The radius of an event horizon is directly proportional to the mass of the black hole. In the process of orbiting each other and merging, the two black holes would radiate away energy in the form of gravitational waves. Since mass is related to energy (\( E = mc^2 \)), the final merged mass (and therefore the radius, which is proportional) would be less than the sum of two black holes masses.
Problem #4

- The very first high-mass stars in the history of the universe did not produce energy through the CNO cycle.
- The CNO cycle is the cycle of reactions by which intermediate- and high-mass stars fuse hydrogen (H) into helium (He). In a high-mass star going through the CNO cycle, due to the hot core temperature, protons can slam into carbon (C), nitrogen (N) and oxygen (O), which act as catalysts for hydrogen fusion, making it proceed at a far higher rate than hydrogen fusion through the proton-proton chain in low-mass stars.
- However, the early universe did not contain C, N and O nuclei; it contained only H and He. It is not until the era of galaxies that generation after generation of star formation in galaxies built elements heavier than H and incorporated them into new star systems.
- Therefore, in the absence of C, N, and O in the early universe, the very first high-mass stars were formed from clouds made only of H and He and could not fuse hydrogen through the CNO cycle.

Problem #5

No elements heavier than carbon (C) would exist if the universe contained only low-mass stars. The core temperatures in low mass stars are not sufficiently hot to fuse carbon nuclei into heavier elements. In fact, carbon fusion is only possible at temperatures above 600 million K. However, degeneracy pressure halts the collapse of a low mass star’s core before it can reach such temperature. When a low-mass star is left with an inert carbon core, it essentially reaches the end of its life; its luminosity and radius increase, and strong convection dredges up carbon from its core, and it is eventually blown into interstellar space with the stellar winds.

Problem #6

(a) The distance d (in parsecs) between two celestial objects, using the parallax angle p, is calculated as follows:

\[ d \text{ (in parsecs)} = \frac{1}{p \text{ (in arcsec)}} \]

Converting to light-years, we have:

\[ 1.351 \text{ parsecs} \times \frac{3.26 \text{ ly}}{1 \text{ parsec}} = 4.405 \text{ ly} \]

\( \Rightarrow \) The distance between the Earth and Alpha Centauri is 4.405 light-years.
(b) \[ d \ (\text{in parsecs}) = \frac{1}{p \ (\text{in parsecs})} = \frac{1}{0.286} = 3.496 \text{ parsecs} \]

Converting to light-years, we have:

\[ 3.496 \text{ parsecs} \times 3.26 \text{ ly/parsec} = 11.40 \text{ ly} \]

The distance between the Earth and Procyon is 11.40 light-years.

**Problem #7**

We want to determine \( m_1 \) and \( m_2 \) and we know from their identical Doppler shifts that \( m_1 = m_2 = m \).

The orbit of a binary system is given by Newton's revision of Kepler's third law:

\[ p^2 = \frac{4\pi^2}{G(m_1+m_2)} \]

\[ \Rightarrow 2m = \frac{4\pi^2}{Gp^2} \ (\star) \]

The circular radius \( a \) is given by:

\[ v = 2\pi a \quad \Rightarrow \quad a = \frac{vp}{2\pi} \ (\star \star) \]

Substituting \((\star \star)\) in \((\star)\), we have:

\[ 2m = \frac{4\pi^2}{Gp^2} \left( \frac{vp}{2\pi} \right)^3 = \frac{pv^3}{2\pi G} \]

\[ \Rightarrow m = \frac{pv^3}{4\pi G} \]

We need to convert the 6-month orbital period into seconds:

\[ \frac{1 \text{ year} \times 365 \text{ days} \times 24 \text{ hrs} \times 60 \text{ min} \times 60 \text{ s}}{2 \text{ year} \times 1 \text{ day} \times 1 \text{ hr} \times 1 \text{ min}} = 1.577 \times 10^7 \text{ s} \]
Substituting the values in the previous expression, we have:

\[ m = \frac{ry^3}{4\pi G} \]

\[ = \frac{(1.577 \times 10^{-3}) (80,000 \text{ m/s})^3}{4 \pi (6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2)} \]

\[ = 9.63 \times 10^{30} \text{ kg} \]

\( \Rightarrow \) The mass of each star is \( 9.63 \times 10^{30} \text{ kg} \).

**Problem #8**

The radius of a star is given by:

\[ r = \frac{1}{\sqrt{4\pi G \sigma T^4}} \]

Substituting the given values, we have:

\[ r = \sqrt{\frac{2G (3.8 \times 10^{26} \text{ watts})}{4\pi (5.7 \times 10^{8} \text{ watt/m}^2 \cdot \text{K}) (9,400 \text{ K})^4}} \]

\[ = 1.3 \times 10^9 \text{ m} \]

\( \Rightarrow \) The radius of Sirius A is \( 1.3 \times 10^9 \text{ m} \).

**Problem #9**

Brown dwarfs are similar to Jovian (Jupiter-like) planets because they are too small to shine with energy generated by nuclear fusion in their core; their mass is below the 0.8 \( M_{\text{Sun}} \), minimum threshold required to be a star. Below such threshold, electron degeneracy pressure hails the gravitational collapse of the brown dwarf before fusion becomes self-sustaining. Moreover, brown dwarfs are stable objects like planets; they do not die like stars because their degeneracy pressure does not diminish with time, such that they can resist the crush of gravity.

On the other hand, brown dwarfs are like stars (more specifically, failed stars) because they form through a similar process as stars (although without sufficient mass to sustain prolonged hydrogen fusion in their core), their surface temperature is relatively high and they emit light (primarily infrared).
Problem #10

The escape velocity is given by the following formula:

\[ V_{\text{escape}} = \sqrt{\frac{2GM}{R}} \]

Substituting the values for the red giant, we have:

\[ V_{\text{escape (red giant)}} = \sqrt{\frac{2GM}{R}} \]

\[ = \sqrt{\frac{2GM_{\text{red}}}{100R_{\text{red}}}} \]

\[ = \sqrt{\frac{2(6.67 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2))(2 \times 10^{30} \text{ kg})}{100(6.95 \times 10^8 \text{ m})}} \]

\[ = 6.2 \times 10^4 \text{ m/s} \]

The escape velocity from the Sun is:

\[ V_{\text{escape (Sun)}} = \sqrt{\frac{2GM_{\text{Sun}}}{R_{\text{Sun}}}} \]

\[ = \sqrt{\frac{2(6.67 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2))(2 \times 10^{30} \text{ kg})}{6.95 \times 10^8 \text{ m}}} \]

\[ = 6.2 \times 10^5 \text{ m/s} \]

Therefore, the escape velocity from the red giant is 10 times less than the escape velocity from the Sun. Given its smaller escape velocity, particles are more likely to escape from the gravitational pull of the red giant, and it has stronger stellar winds than our Sun. \( \checkmark \)