READING and PROBLEM SET 9

READING: Textbook, Chapter 8, Sections 8.5 - 8.8, Chapter 9 (optional)

PROBLEMS, due TUESDAY, April 10 2018 (in class):

1. In class I discussed the Shapiro time delay. Starting point was a sketch showing
the trajectories of light in flat space-time and in curved space-time. I then sketched the
computation of the time delay. As I mentioned, there is an inconsistency between the
computation and the sketch. What is this inconsistency? Try to fix the error, and show
that to leading order in $m$, the results of the improved calculation agree with what was
obtained in class.

2. Our current universe appears to be dominated by a cosmological constant. Compute
the lifetime of our universe assuming that today (when the Hubble expansion rate is
$h \times 100\text{km s}^{-1}\text{Mpc}^{-1}$ with $h \simeq 0.7$) 70% of the energy is in the form of the cosmological
constant and 30% is in the form of cold matter.

3. In class I wrote down the equation of motion for a scalar field in an expanding
flat FRW Universe. I also mentioned what is meant by the slow rolling approximation.
Consider now a homogeneous scalar field in the chaotic inflation model with potential

$$V(\phi) = \frac{1}{4} \lambda \phi^4.$$ 

Show that for sufficiently large values of $\phi$, the slow rolling equation is self consistent and
find the limiting value of $\phi$ for which this ceases to be true.

4. Textbook, Problem 8.5

5/6. Consider a contracting matter-dominated universe which tends to a Big Crunch
singularity at time $t = 0$.

a) Plot the time evolution of the Hubble radius, the particle horizon and the wavelength
of a fixed comoving scale.

b) Consider vacuum initial conditions for cosmological fluctuations and compute the power
spectrum of the fluctuations on super-Hubble scales close to the bounce point.

c) Compare with the predictions of inflationary cosmology.

NB: Attempt only after the lecture of April 3.