1/2. Consider the Mathieu equation discussed in class

\[ \ddot{\chi}_k + (k^2 + g^2 \sigma^2 \cos(mt))\chi_k = 0, \]

where \( \sigma \) is a mass scale, \( g \ll 1 \) is a dimensionless coupling constant and \( m \) is a frequency. Find the resonance bands of the system, i.e. the values of \( k \) for which the equation has exponentially increasing solutions.

3. For an oscillating inflaton \( \varphi \) background (in the context of large field inflation), the equation of motion for a massless field \( \chi \) coupled to \( \varphi \) as discussed in class is an equation of Mathieu type with a very large coupling constant. In this case there is broad parametric resonance. Using the adiabaticity condition discussed in class, determine the range of \( k \) values for which there is resonance.

4. In the above case, verify that there is a range of \( k \) values for which the expansion of space can be neglected.

5/6. In class I mentioned the tachyonic resonance which appears if the field \( \chi \) is coupled to \( \varphi \) with a negative coupling constant, i.e. the interaction Lagrangian is

\[ \mathcal{L}_I = \frac{1}{2} g \varphi^2 \chi^2, \]

where \( g \) is a positive constant. In order that the system is stable, one needs to assume the presence of a nonlinear term \( \lambda \chi^4 \) in the potential for \( \chi \). Assume that \( \lambda \) is a very small positive constant. Study the growth of fluctuations of \( \chi \) in this model (neglecting the expansion of space), discuss what back-reaction effects need to be considered, and estimate how long the resonance of \( \chi \) persists until back-reaction effects become important.