Muon identification with VERITAS using the Hough Transform

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DEDICATION

This thesis is dedicated to my mother, Heather Ayerst.
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I would like to thank my supervisor, Ken Ragan for suggesting the topic of this thesis and for all his good advice and feedback. I would also like to thank David Hanna, Andrew McCann, Michael McCutcheon, Simon Archambault, Sean Griffin, David Staszak, Gordana Tesic, Gernot Maier, Roxanne Guenette, and Audrey MacLeod for their insight and suggestions. Thanks to Anna O’Faoláin de Bhróithe for help with comparing the technique described in this thesis with the standard technique. Special thanks to Paul Mercure for providing technical assistance with the computer network.
ABSTRACT

Imaging atmospheric Cherenkov telescope (IACT) arrays such as VERITAS are used to perform very high-energy gamma-ray astronomy. This is accomplished by detecting and analyzing the Cherenkov light produced by gamma-ray initiated atmospheric air showers. IACTs also detect the Cherenkov light produced by individual muons. The Cherenkov light produced by muons is well understood, and can be used as a calibrated light source for the telescopes. Muons create characteristic annular patterns in the cameras of IACTs, which may be identified using parametrization algorithms. One such algorithm, the Hough transform, has been used to identify muons in VERITAS data. The details of the Hough transform and its implementation on VERITAS data will be described, as well as the use of parameters derived from the Hough transform for muon identification. In addition, the selection of muon rings appropriate for calibration purposes will be described. Finally, the Hough transform-based muon selection technique will be compared to the standard VERITAS muon selection technique.
ABRÉGÉ

Les systèmes de télescopes par imagerie Cherenkov tel que VERITAS sont utilisés pour l’astronomie à rayons gammas de très hautes énergies. Ceci est accompli par la détection et l’analyse de la lumière Cherenkov produite par les gerbes de particules causées par l’interaction des rayons gammas avec l’atmosphère. Ces télescopes détectent aussi la lumière Cherenkov produite par les muons. La lumière Cherenkov produite par les muons est bien comprise, et peut être utilisée comme source de calibration pour les télescopes. Les muons forment un anneau dans leur caméra, et peuvent être identifiés en utilisant des algorithmes de paramétrisation. La transformée de Hough est un de ces algorithmes, et a été utilisé afin d’identifier les muons dans les données de VERITAS. Les détails de la transformée de Hough et son application avec VERITAS seront présentés, ainsi que l’utilisation des paramètres en découant pour l’identification de muons. De plus, la sélection d’anneaux de muons appropriés pour des besoins de calibration sera décrite. Finalement, la technique de sélection de muons basé sur les transformées de Hough sera comparée à la technique de sélection de muons standard de VERITAS.
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CHAPTER 1
Introduction

1.1 Thesis overview

This thesis describes the identification of circular patterns produced by atmospheric muons in VERITAS (Very Energetic Radiation Telescope Array System) data. This is accomplished by the use of parameters derived from an algorithm called the Hough transform. The selection of muon rings suitable for calibration purposes is also described. Finally, the Hough transform-based muon selection technique described in this thesis is compared to the standard VERITAS muon selection technique.

Chapter 2 describes ground-based gamma-ray astronomy, the imaging atmospheric Cherenkov technique and the VERITAS experiment. The VERITAS energy calibration procedure is also briefly described.

Chapter 3 describes the origin and geometry of muon events as observed by VERITAS, the motivation for using single telescope data and the use of muon rings for calibration purposes.
Chapter 4 describes the Hough transform algorithm using lines and circles as examples, as well as the implementation of the circular Hough transform on VERITAS data.

Chapter 5 describes the motivation for, and use of parameters derived from the Hough transform for muon identification. The results of applying cuts on these parameters to VERITAS data is also described.

Chapter 6 describes the selection of muon rings suitable for calibration purposes using the Hough transform and a new parameter created for this purpose.

Chapter 7 compares the Hough transform-based muon selection technique described in this thesis with the standard VERITAS muon selection technique.

Chapter 8 provides a summary of the results obtained and describes possible future work.

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2.1  Ground based gamma-ray astronomy

The electromagnetic spectrum includes radio, infrared light, visible light, ultra-violet light, x-rays and gamma rays. Gamma rays are defined to be photons with energies of 100 keV and above, and are the most energetic photons of the electromagnetic spectrum. Gamma-ray astronomy involves the detection and analysis of gamma-ray photons from astrophysical sources such as: active galactic nuclei (AGN), supernova remnants (SNR), pulsars, globular clusters, and x-ray binaries [3]. Unlike photons of lower energies, gamma-ray photons cannot be reflected or refracted using traditional optics. Therefore, special techniques must be used in order to detect them. Below approximately 30 GeV, gamma rays can be detected directly using balloon or space based telescopes such as the Fermi gamma-ray space telescope [3]. At higher energies, gamma rays are not detected in sufficient quantities by these instruments due to their limited effective areas and the decreasing astrophysical gamma-ray flux with increasing energy. Larger effective areas are required to perform practical gamma-ray astronomy at these energies. Therefore, a ground-based technique (described in section 2.3) is employed, using the atmosphere as part of the detector. This technique, the imaging atmospheric Cherenkov technique, achieves effective areas of approximately $10^5$ m$^2$ [3]. VERITAS, the subject of this thesis, uses this technique and will be described in detail in this chapter.
2.2 An overview of the VERITAS instrument

VERITAS is a ground-based, very high-energy gamma-ray observatory consisting of an array of four imaging atmospheric Cherenkov telescopes (IACTs) located in southern Arizona USA [11]. VERITAS detects the Cherenkov light emitted by atmospheric particle cascades called extensive air showers initiated by very high-energy (VHE) gamma rays in the 50 GeV to 50 TeV energy range. The analysis of the properties of the detected Cherenkov light allows the reconstruction of the arrival directions and energies of the gamma rays, allowing VHE gamma-ray astronomy to be performed. This technique is called the imaging atmospheric Cherenkov technique, and will be described in greater detail in section 2.3. A photograph of the VERITAS array is shown in figure 2–1.

The VERITAS telescopes use a commercial altitude-over-azimuth positioner which supports a reflector, a camera, quadrupod arms and a counterweight. Each software-controlled positioner is able to track a source with a precision better than
0.01 degrees, and can slew at a rate of approximately 1 degree per second. The positioner of a given telescope is connected to a steel optical support structure (OSS) to which 345 hexagonal mirror facets are mounted, creating a segmented spherical reflector (a Davies-Cotton design [4]) with a 12 meter diameter and focal length. The hexagonal mirror facets consist of glass shaped to have a spherical radius of curvature of approximately 24 meters, to which a reflective aluminum coating has been applied. The mirror facets are mounted on triangular frames attached to the OSS using three screws, allowing the mirrors of the reflector to be aligned. The OSS is attached to a counterweight (located behind the OSS) and the quadrupod arms, which are used to keep the camera at the focal point of the reflector. The reflector focuses light from specific angles onto unique positions in the camera plane. Specifically, the relationship between angular separations and distances in the camera plane is called the “plate scale”, and is approximately 207 mm per degree for the VERITAS telescopes. For this reason, angular separations and distances in the camera plane will be used interchangeably in this thesis.

Each camera consists of 499 light-sensitive photomultiplier tubes (PMTs) arranged in a hexagonal lattice. The PMTs consist of glass vacuum tubes that have a light sensitive metal coating called the photocathode applied to the inner front surface. Photons incident on the photocathode cause the emission of an electron (called a photoelectron) with a certain probability called the quantum efficiency. The photoelectron is amplified via the phenomenon of secondary emission, using a series of dynodes kept at different voltages. This results in an amplification of the original photoelectron by a factor of approximately $10^5$, allowing the signal to be
measured [16]. Each PMT generates a signal proportional to the number of photons incident on the photocathode, and therefore can be thought of as a pixel. Each PMT in the VERITAS cameras covers an angle of 0.15 degrees, creating cameras with 3.5 degree fields of view. A photograph of one of the VERITAS cameras is shown in figure 2–2.

2.3 The imaging atmospheric Cherenkov technique

As mentioned previously, VERITAS uses the imaging atmospheric Cherenkov technique to perform gamma-ray astronomy. Astrophysical VHE gamma rays interact with the atmosphere via electromagnetic interactions, producing particle cascades called electromagnetic air showers (illustrated in figure 2–3). Gamma rays produce an electron-positron pair in the vicinity of an atmospheric atomic nucleus, which
produces further gamma rays via bremsstrahlung radiation. These processes repeat, resulting in the production of many electrons, positrons and gamma rays. The air shower develops until the energy of the particles is insufficient to produce further electron-positron pairs. The particles produced in the air showers are typically traveling faster than the speed of light in air, causing the emission of Cherenkov light as they propagate. More information regarding Cherenkov radiation principles can be found in [7]. The Cherenkov light produced in the air showers propagates to the ground, where it can be detected by IACTs, as shown in figure 2–4. The reflectors of the telescopes focus the Cherenkov light onto the cameras, where it is detected by the PMTs. The signals from the PMTs are converted to digital information and stored using a data acquisition system. The data is then analyzed using offline analysis software. The VERITAS data acquisition system will be described in section 2.4 and the VERITAS offline analysis software will be described in section 2.5.

2.4 The VERITAS data acquisition system

The VERITAS PMTs produce analog signals proportional to the number of incident photons, which are digitized and placed in a buffer using flash analog to digital converters (FADCs). The FADCs operate at a sampling rate of 500 MHz and use an 8 bit dynamic range [16]. Signals above a certain threshold are reduced in gain in order to avoid exceeding the maximum signal that the FADCs can digitize. The amount that the gain is reduced in these circumstances (the high/low gain ratio) must be taken into account when analyzing the signal from the PMT.

If the digitized signals from the FADCs were recorded continuously, the files that would be created would consist of mostly uninteresting data, and would require
impractical amounts of storage space. In order to address this issue, VERITAS uses a three-level trigger system that stops the FADCs and records the contents of the buffers over a specified time window when the trigger criteria are met.

The first level of the trigger (L1) requires that a signal from a given PMT channel exceed a specified threshold. When this occurs for a given PMT, that PMT is said to have passed the L1 trigger condition. The L1 trigger ensures that data is recorded only when pixels in the VERITAS cameras detect more than a specified amount of light. For this reason, the L1 trigger is called the pixel trigger. The L1 triggers are implemented by using constant fraction discriminators (CFDs), that ensure that the timing of the L1 trigger is independent of the signal intensity. The PMTs that pass the L1 condition can be thought of as forming binary images, with PMTs that pass
the L1 condition assigned a value of one, and the PMTs that do not pass the L1 condition assigned a value of zero. These images are also called “binary data”, “L1 trigger patterns” or “CFD trigger patterns”.

The second level of the trigger (L2) requires that a configurable number\(^1\) of contiguous pixels of a given telescope pass the L1 condition. When this condition occurs for a given telescope, that telescope is said to have passed the L2 trigger condition. The L2 trigger ensures that data is recorded when certain patterns of pixels passing the L1 condition are present in the camera, reducing the amount of data recorded.

\(^1\) The default value is three, with a maximum of five.
data recorded due to night sky background fluctuations. For this reason, the L2 trigger is called the pattern trigger.

The third level of the trigger requires that multiple telescopes pass the L2 trigger condition. When this condition occurs, the array is said to have passed the L3 trigger condition. For this reason, the L3 trigger is also called the array trigger. The L3 trigger ensures that the recorded data is of high enough quality for offline analysis (described later in this section). The L3 trigger also removes much of the background due to distant muons, which will be described in greater detail in section 3.3.

When the L3 condition is met, the data from the FADC buffers of a given telescope is sent to a computer called an event builder, where it is combined into a “telescope event”. A telescope event consists of the digitized values of each PMT signal for a given telescope, sampled in 2 ns increments over a range of 20 samples (FADC traces), a list of which PMTs passed the L1 condition and other information. The telescope events for each telescope are then sent to a computer called the harvester, where they are combined into a four-telescope “array event”. The array events of a given run are assigned a unique number, combined and saved to disk in a “Compressed VERITAS Bank Format” (CVBF) file, the standard VERITAS file format [16]. The CVBF file is sent to an archive where it is made available to members of the VERITAS collaboration for offline analysis (described in section 2.5).

2.5 The VERITAS offline analysis software

The offline analysis of VERITAS data is implemented in two separate offline analysis software packages: VEGAS (VEritas Gamma ray Analysis Suite) and Event
Display. The VEGAS offline analysis is performed in a series of “stages”, and is similar to the procedure used in Event Display.

Stage one of the offline analysis performs the calculations necessary for timing corrections and flat fielding of the FADC traces, using special calibration runs where the camera is uniformly illuminated with light from pulsed LEDs (flasher runs). Stage one also obtains various quantities stored in the VERITAS database, such as pointing and other calibration information.

Stage two of the offline analysis applies the timing corrections and flat fielding information calculated in stage one to the FADC traces. When the corrections have been implemented, the FADC traces are integrated (summed) within a specified time window, resulting in an intensity value associated with each pixel in the cameras. The intensity values that would be calculated for each pixel in the absence of light (calculated in stage 1) are subtracted from the intensity values of each pixel (pedestal subtraction), resulting in an intensity value assigned to each pixel in the cameras, called the charge.

Stage three of the offline analysis sets the charge of pixels that do not reach specified charge thresholds to zero, a process called “cleaning”. Cleaning removes pixels from the image that are likely due to noise. The resulting integrated, pedestal-subtracted and cleaned images are called “pulse height data”, “pixel intensity patterns” or simply images with charge or pixel intensity information. The cleaned and pedestal-subtracted pixel intensity patterns of each telescope are then analyzed to obtain a set of parameters (called Hillas parameters) for each telescope, which are
used in subsequent stages of the analysis. An important parameter called \textit{size} is calculated for each telescope at this stage and corresponds to the summed \textit{charge} of all of the cleaned and pedestal-subtracted pixels for that telescope. The energy calibration of the telescopes (described in sections 2.6 and 3.4) requires an understanding of the relationship between the \textit{size} recorded by a given telescope and the number of photons incident on the reflector. The other Hillas parameters calculated in stage three are determined from the geometry of the shower images in the cameras.

Stage four of the offline analysis uses the Hillas parameters derived in stage three to determine the properties of the shower (and hence the initiating particle), a process called “reconstruction”. Specifically, parameters derived from the geometry of the images in each telescope are combined and used to determine the arrival direction of the shower in the sky, as well as the location where the shower axis intersects with the ground (the “core distance”, “shower impact distance” or “impact parameter”). In addition, the \textit{size} values recorded by each telescope are combined and used to estimate the energy of the shower, using lookup tables created from simulations that use calibration measurements which will be described in section 2.6. Stage four also uses the Hillas parameters derived in stage three to calculate “array parameters” that are used in stage five.

Stage five uses the array parameters calculated in stage four to discriminate between showers caused by gamma rays and the (far more numerous) showers caused by cosmic rays (described in section 3.1). The showers that appear to be caused by gamma rays are kept for stage six and the showers that appear to be caused by cosmic rays are discarded.
Stage six of the offline analysis uses the properties of the events classified as gamma rays to produce sky maps and calculate the statistical significance of signals, the end result of the IACT technique [16].

2.6 Energy calibration

As mentioned previously, the energies of the gamma rays detected by VERITAS are estimated by using lookup tables produced from simulations. The simulations create the lookup tables by simulating gamma rays of various energies, and determining what parameter values would be measured by a simulated VERITAS array [16]. The energy of the gamma rays is related to the number of Cherenkov photons emitted by the electromagnetic shower, and therefore the number of photons incident on the reflector of a given telescope. In order to create the lookup tables from the simulations, the relationship between the number of photons incident on the reflector and the size (described in section 2.3) recorded by a given telescope must be determined. This is accomplished by combining several calibration measurements, including reflectivity measurements, PMT quantum efficiency measurements and measurements of the number of digital counts recorded by the FADCs due to a single photoelectron [6] [8]. These measurements can be verified by the analysis of muon images, which will be described in Chapter 3.
CHAPTER 3
Muons

3.1 The origin of muon events

Muons (elementary charged particles with many of the properties of the electron, but with greater mass) are produced in the atmosphere in hadronic air showers, a schematic diagram of which is illustrated in figure 3–1. Hadronic air showers are caused by interactions of high-energy cosmic-ray particles with the atmosphere, which produce charged and neutral pion (pion) particles ($\pi^0$, $\pi^+$, $\pi^-$) as well as nuclei. The neutral pions decay into gamma rays via $\pi^0 \rightarrow \gamma + \gamma$, initiating electromagnetic sub-showers (discussed in section 2.3), and the charged pions decay into neutrinos and muons via $\pi^+ \rightarrow \mu^+ + \nu_\mu$ and $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. Muons can travel large distances through the atmosphere, many reaching ground level before decaying via $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ and $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$. The muons produced in hadronic showers are typically relativistic particles, emitting Cherenkov light as they propagate through the atmosphere, which can be detected by the VERITAS telescopes. The geometry of the PMT patterns that appear in the VERITAS cameras due to the Cherenkov light emitted by muons will be described in the next section.

3.2 The geometry of muon events

As mentioned previously, The VERITAS reflectors focus light from specific angles of incidence (within the 3.5 degree field of view) onto unique positions in the camera plane. Particular angular separations correspond to specific distances in the
Figure 3–1: A schematic diagram of a hadronic shower. Figure modified from [12].

Figure 3–2: Muon event geometry for a muon hitting the reflector, parallel to the optical axis. Figure modified from [6].
camera plane. Relativistic muons undergo the emission of Cherenkov light in a cone at a nearly constant angle (the Cherenkov angle) from their axis of propagation as they travel through the atmosphere. The Cherenkov photons emitted at a given point trace out a cone as they propagate from their point of origin toward the ground, as illustrated in the 3D view of figure 3–2. Each photon path on the cone corresponds to one possible angle of incidence at the reflector, and is focused onto a unique position on the camera. Each possible angle of incidence of the photons is offset from the angle of incidence of the muon by its Cherenkov angle for all azimuthal angles. Therefore, the Cherenkov light emitted by muons is focused into circular (as opposed to elliptical) patterns called rings in the cameras of IACTs, as shown in the camera plane of figure 3–2. The location of the center of the ring in the camera plane corresponds to the angle of incidence of the muon with respect to the optical axis of the telescope, and the radius of the ring corresponds to the Cherenkov angle of the muon. The reflector plane in figure 3–2 shows the impact point of the muon on the reflector plane, where \( b \) is the impact parameter of the muon, \( D/2 \) is the radius of the reflector and \( \varphi \) is the azimuthal angle. \( \rho(b, \varphi) \) represents the distance from the impact point of the muon to the edge of the reflector, and is proportional to the amount of light received from a given azimuthal angle. Muon rings with azimuthally symmetric light intensity levels are produced in the camera plane only if the reflector receives the same amount of light for all azimuthal angles, corresponding to muons that fall directly onto the center of the reflector \( (b = 0) \). Azimuthally complete (closed) rings are produced in the camera plane if the reflector receives light for all azimuthal angles, corresponding to muons that fall on the reflector, i.e: \( b < D/2, \)

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Figure 3–3: A muon event recorded by VERITAS. The color of the pixels represents the number of integrated digital counts (charge) recorded for each PMT, with blue representing the lowest charge and yellow representing the highest charge. Figure taken from [6].

Azimuthally incomplete rings (arcs) are produced in the camera plane if the reflector receives light over a limited range of azimuthal angles, corresponding to muons that fall outside the reflector, ie: $b > D/2$. Figure 3–3 shows the circular pixel intensity pattern (before pedestal subtraction and cleaning) produced in a VERITAS camera by a muon with $b < D/2$. The color of the pixels represents the number of integrated digital counts (charge) recorded for each PMT, with blue representing the lowest charge and yellow representing the highest charge. Since muons produce characteristic circular PMT patterns, they can be identified using shape parametrization algorithms such as the Hough transform, which will be described in chapter 4.
3.3 Single telescope data

Muons that appear as full rings in one telescope will appear as small arcs in another telescope, as shown in figure 3–4. The vertical view shows the path of the muon (the dashed black line) as it falls directly onto the middle of the reflector of T1. The dashed red lines represent the paths of the Cherenkov photons. The overhead view shows that T1 receives light over an azimuthal angle of 360 degrees, but T2 receives light only over a small angle $\Psi$. The camera views show the resulting light distributions in the camera plane of each telescope. A complete 360 degree ring is formed in the camera plane of T1, whereas a small arc of angular extent $\Psi$ appears in the camera plane of T2. Small arcs may not cause the L2 trigger condition (discussed in section 2.3) to be met for a given telescope. This effect significantly reduces the number of muons that pass the L3 trigger condition (discussed in section 2.3) for multiple telescope runs compared to the number of muon events that pass the L2 trigger condition for a given telescope. For this reason, the data discussed in this thesis consists of single telescope runs with events recorded when the L2 trigger condition for that telescope is met.
3.4 Muons as calibrated light sources

The detection of muon events with VERITAS is motivated by the fact that muons consist of calibrated Cherenkov light sources, which will be briefly described in this section. The summed charge recorded by the hit PMTs for a gamma ray event (size) is proportional to the number of photons incident on the reflector from the air shower, which is in turn related to the energy of the gamma ray that initiated it, as described in section 2.6. The relationship between the size of an event recorded by VERITAS and the number of photons incident on the reflector is normally calculated by combining several calibration procedures (described in section 2.6). Despite such calibration procedures, this relationship is poorly constrained.Muon data can be used to verify the results of these calibration procedures. Specifically, the number of photons incident on the reflectors for a muon event can be calculated given the Cherenkov angle and impact parameter of the muon, both of which can be determined by the analysis of the muon ring image formed in the camera. The calculated number of photons incident on the reflectors can then be related to the size of the event. This procedure is effective for events with just one muon ring in the image due to the difficulty associated with analyzing events with multiple rings. In addition, events with multiple rings in the image occur very infrequently. Therefore, the muon identification procedures described in this thesis are used to identify events with just one muon ring in the image. Further details of the calibration of IACTs with muons are described in [15], [13] and [6]. Muon calibration analysis with single telescope data is currently underway using the VERITAS offline analysis packages. In addition, the development of dedicated hardware muon triggers for each telescope
is being considered in order to provide a sample of muons for calibration purposes during normal operations. The Hough transform (described in chapter 4) is being considered for use as a component of the the muon selection algorithms of these calibration efforts.
4.1 A description of the Hough transform

The Hough transform is an algorithm used for parametrizing given shapes in digital images. The algorithm was first published in 1962 in a patent by Paul V. C. Hough [10], and has since become a standard image analysis tool [1] [9]. Mathematically, the Hough transform has been shown to be equivalent to the radon transform, both of which are equivalent to a process called template matching [17].

The Hough transform finds the best parameters for an assumed shape in a digital image using a voting procedure implemented with a quantized parameter space, instead of iteratively fitting an assumed shape in image space. The quantized parameter space is represented by a structure called an accumulator array, which allows the accumulation of votes in the parameter space. The accumulator array can be implemented with any data structure that can store votes in a quantized parameter space, such as an array, a vector or a histogram and is constructed so that each possible parametrization of the assumed shape has a unique bin in the accumulator array.

Once the parameter space is constructed, the voting procedure can be implemented. Each possible parametrization of the assumed shape will intersect a certain number of pixels in the image. Therefore, each pixel in a digital image can be thought of as being associated with a finite number of possible parametrized shapes.
The Hough transform is implemented by adding the intensity values of each pixel in the image to the bins of the accumulator array corresponding to the possible parametrized shapes that pass through those pixels. Once the voting procedure has been completed, the coordinates of the center of the bin of the accumulator array with the most votes corresponds to the best parametrization of the assumed shape in the image. The next two sections will describe the implementation of the Hough transform algorithm using specific shapes (lines and circles) as examples.

### 4.2 The parametrization of lines using the Hough transform

A line may be parametrized using two parameters: $r$ and $\theta$, where $r$ is the distance from the origin to the closest point on the line, and $\theta$ is the angle between the normal vector of the line and the $x$ axis, as shown in figure 4–1. The use of the $(r, \theta)$ parametrization for implementing the Hough transform was first described in [5]
and is used in order to avoid the unbounded parameter space associated with a slope-intercept parametrization. In order to implement the Hough transform for lines, a 2-dimensional \((r, \theta)\) parameter space is constructed using an accumulator array. The co-ordinates of the center of each bin of the accumulator array correspond to a possible \((r, \theta)\) parametrization of a line. The linear Hough transform is implemented by adding the intensity values of each pixel in the image to the bins of the accumulator array that correspond to the lines that pass through those pixels. Following the voting procedure, the \((r, \theta)\) co-ordinates of the center of the bin of the accumulator array with the most votes correspond to the best linear parametrization of the image. Figure 4–2 shows a binary image of two lines with noise (left), and the corresponding \((r, \theta)\) parameter space (right). An individual pixel in the image gives rise to a sinusoid
in the parameter space. The two concentrations (the two bright areas) visible in the parameter space correspond to points of overlap of the sinusoids, and represent the best \((r, \theta)\) parametrizations of the lines in the image. The locations of these peaks in the parameter space are clearly well defined despite the noise pixels present in the image.

4.3 The parametrization of circles using the Hough transform

Muons produce circular patterns in the images recorded by VERITAS, as described in section 3.2. Therefore, the circular Hough transform must be used for parametrizing these images. This technique was described, and shown to be effective at parametrizing circles in [14], which provided motivation for implementing the circular Hough transform with VERITAS data. A circle can be parametrized using three parameters: the coordinates of the center of the circle, \((x, y)\) and the radius.
Figure 4–4: Pixel pattern (upper left) and 2D parameter space projections for a circular Hough Transform. Figure modified from [14].
of the circle, \( r \) as shown in figure 4–3. In order to implement the circular Hough transform, a 3 dimensional \((x, y, r)\) parameter space is constructed using an accumulator array. The co-ordinates of the center of each bin of the accumulator array correspond to a possible \((x, y, r)\) parametrization of a circle. The circular Hough transform is implemented by adding the intensity values of each pixel in the image to the bins of the accumulator array that correspond to the circles that pass though those pixels. The \((x, y, r)\) coordinates corresponding to the center of the bin of the accumulator array with the most votes is taken to be the best circular parametrization of the image. Figure 4–4 shows an image of a circular pixel pattern with noise (upper left), and corresponding 2D parameter space projections of the accumulator array. The upper right image shows the \((x, y)\) plane for best \( r \), the lower left image shows the \((y, r)\) plane for best \( x \) and the lower right image shows the \((x, r)\) plane for best \( y \). Again, the the location of the peak in the parameter space is well defined and insensitive to the noise present in the image.

4.4 Implementing the circular Hough transform with VERITAS data

In order to implement the circular Hough transform with VERITAS data, a lookup table was constructed consisting of a list of circle parametrizations associated with each of the 499 pixels in the VERITAS cameras. The lookup table was constructed by determining which pixels were associated with particular parametrized circles, and was implemented with a ROOT tree data structure. The locations of the centers of each PMT in the VERITAS cameras were used as the location of the centers of the circles. The radii of the circles consisted of values from 3 PMT diameters to 11 PMT diameters, incremented by a third of a PMT diameter. This
choice of possible circle parametrizations resulted in 12475 distinct circles, which were used to generate the lookup table. The condition for a pixel to be associated with a particular circle was:

\[ | D_{\mu p} - R_{\mu} | \leq R_{PMT} \]

Where \( D_{\mu p} \) is the distance from the center of the circle to the center of the pixel, \( R_{\mu} \) is the radius of the circle, and \( R_{PMT} \) is the radius of a PMT.

If a circle was found to be associated with a particular pixel, the \((x, y, r)\) parametrization of that circle was added to the list of circle parametrizations associated with that pixel. Figure 4–5 shows the pixels associated with different circle parametrizations.
parametrizations that were used to generate the lookup table. This procedure was implemented for all 12475 circles, resulting in a list of \((x, y, r)\) circle parametrizations associated with each of the 499 pixels in the VERITAS cameras. The construction of the lookup table allowed a direct implementation of the Hough transform algorithm with VERITAS data.

Following the construction of the lookup table, an accumulator array was constructed using a 3 dimensional histogram, again in the ROOT framework. The quantization of the parameters \(x\), \(y\), and \(r\) in the parameter space was matched to the quantization of the parameters of the circles that were used to generate the lookup table. Specifically, the locations of the centers of the bins on each axis of the accumulator array correspond to the possible parameter values of the circles that were used to generate the lookup table. Matching the bins of the accumulator array with the circle parametrizations in the lookup table allowed each pixel of the VERITAS cameras to be associated with the bins of the accumulator array whose \((x, y, r)\) parametrizations describe circles that intersect the pixels in question. For images with charge information (discussed in section 2.3), the Hough transform was calculated by adding the charge of each of the hit pixels in the image to the bins of the accumulator array associated with those pixels. In the case of binary data (discussed in section 2.3), one vote per hit pixel was added to the bins of the accumulator array associated with those pixels. In both cases, the coordinates of the center of the bin of the accumulator array with the most votes correspond to the best parametrization of the event.
Figures 4–6 and 4–7 show the pixel intensity patterns (upper left) and 2D parameter space projections for a muon (Figure 4–6) and a non-muon (Figure 4–7) event from run 47511 (a 15 minute single telescope run taken with T3 in October 2009). The coordinates of the center of the bin of the accumulator array with the most votes was taken to be the best \((x, y, r)\) parametrization of the event, and was used to produce the 2D parameter space projections. The upper right plots show the \((x, y)\) plane for best \(r\), the lower left plots show the \((x, r)\) plane for best \(y\) and the lower right plots show the \((y, r)\) plane for best \(x\). The red, green and blue circles superimposed over the pixel intensity patterns correspond to the best, second best and third best parametrizations of the event, and correspond to the bins of the accumulator array with the highest, second highest and third highest values. For the muon event (figure 4–6), the parametrization of the circular pixel pattern is well defined, as the three best parametrizations trace the pixel pattern quite well. This can also be seen from the sharp peaks in the parameter space projections. For the non-muon event (figure 4–7), the parametrization of the non-circular pixel pattern is not very well defined, as the three best parametrizations differ significantly in center location and radii. The parameter space projections also have broader (less peaked) distributions. The features of the three best parametrizations of the pixel intensity patterns as well as the parameter space projections were used to motivate variables for muon identification, which will be described in Chapter 5.
Figure 4–6: Pixel pattern and 2D parameter space projections for a muon event. The
colour of the pixels represents the charge, with blue representing the lowest charge
and red representing the highest charge. The red, green and blue circles represent
the bins of the accumulator array with the highest, second highest and third highest
values. The height, as well as the colours of the bins represent the number of votes
in the bin.
Figure 4–7: Pixel pattern (upper left) and 2D parameter space projections for a non-muon event. The colour of the pixels represents the *charge*, with blue representing the lowest *charge* and red representing the highest *charge*. The red, green and blue circles represent the bins of the accumulator array with the highest, second highest and third highest values. The height, as well as the colours of the bins represent the number of votes in the bin.
CHAPTER 5
Muon identification using the Hough transform

5.1 Muon identification by eye

In order to test the effectiveness of muon identification using the Hough transform, many events in run 47511 were visually inspected and categorized. The events were labeled muon events, non-muon events and ambiguous events, using software designed for this task. The software displayed an image of the pixel intensity pattern (see section 2.3) for a particular event, and prompted the user to assign one of the previously mentioned categories to the event. The categorization was performed and saved in a ROOT tree for later use. An event was categorized as a muon event if a circular pattern was obviously present in the image, and categorized as a non-muon event if there was no clear circular pattern present. An event was categorized as ambiguous if it was not clear if a circular pattern was present, or if additional features were present in addition to a circular component. Figures 5–1, 5–2 and 5–3 show examples of each type of event. The visual scan covered event numbers from 1 to 81682, corresponding to 22774 events or approximately 4.4 minutes of data taking. Events with fewer than 10 hit pixels were excluded from the analysis. This was done because circular patterns are difficult to identify in images consisting of fewer than 10 hit pixels. Of the 22774 events that were viewed and categorized, 1516 were categorized as muon events, 17027 were categorized as non-muon events and 4231 events were categorized as ambiguous.
Figure 5–1: Examples of events visually categorized as muons. The colour of the pixels represents the charge, with blue representing the lowest charge and red representing the highest charge.

Figure 5–2: Examples of events visually categorized as non-muons. The colour of the pixels represents the charge, with blue representing the lowest charge and red representing the highest charge.
Figure 5–3: Examples of events visually categorized as ambiguous. The colour of the pixels represents the charge, with blue representing the lowest charge and red representing the highest charge.

5.2 Variables for muon identification

5.2.1 The number of hit pixels ($N_{pix}$)

The number of hit pixels ($N_{pix}$) in the event was used as a first cut for muon identification. The meaning of this number varies between different representations of the events. For data with pixel intensity information, the number of hit pixels corresponds to the number of pixels with non-zero charge values that remain in the image after pedestal subtraction and cleaning (as discussed in 2.5). For images consisting of the L1 trigger pattern, the number of hit pixels corresponds to the number of pixels that received enough light to trigger the CFDs. Only events (for both types of data) with 10 hit pixels or more ($N_{pix} \geq 10$) were analyzed in this
work. This was again motivated by the fact that images with less than 10 hit pixels are difficult to categorize.

5.2.2 The $AP$ parameter

For the muon event shown in figure 4–6, sharp peaks are seen in the corresponding projections of the accumulator array. For the non-muon event shown in figure 4–7, a broader distribution is observed. This observation was used to motivate a parameter called $AP$ (accumulator peakedness) for the purpose of muon identification. This parameter consists of the value of the bin of the accumulator array with the most votes divided by the average non-zero bin value of the accumulator array. Specifically:

$$AP = \frac{\text{Largest bin value}}{\left(\frac{\text{Sum of all bin values}}{\text{Number of non-zero bins}}\right)}$$

$AP$ can be thought of as a measure of the “strength” or “signal to noise ratio” of the best parametrization of the event. Since muon events produce sharp peaks in the accumulator array, they should have large $AP$ values. The $AP$ parameter is partially sensitive to noise, as muon events with significant amounts of noise tend to have lower $AP$ values on average than muon events without noise. Despite the noise sensitivity, muon events with significant amounts of noise have been identified using the $AP$ parameter.
5.2.3 The $TD$ parameter

The three circles superimposed over each event in figures 4–6 and 4–7 are the three best parametrizations of the pixel pattern, and correspond to the bins of the accumulator array with the highest three values. The three best parametrizations are close together in $(x, y, r)$ space for the muon event, and farther apart for the non-muon event. This is also seen in the parameter space projections of each event, as the bins with the highest values are clustered together for the muon event and widely spaced for the non-muon event. This observation was used to motivate another accumulator array-derived parameter called $TD$ (total distance), for the purpose of muon identification. This parameter consists of the total hyper-distance in parameter space defined by the three best parametrizations of the event. Specifically, if $(x_1, y_1, r_1)$, $(x_2, y_2, r_2)$ and $(x_3, y_3, r_3)$ represent the best, second best and third best parametrizations of the event, then:

$$TD = D_1 + D_2 + D_3$$

where,

$$D_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (r_1 - r_2)^2}$$
$$D_2 = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2 + (r_1 - r_3)^2}$$
$$D_3 = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2 + (r_2 - r_3)^2}$$

$TD$ can be thought of as a measure of the unanimity of the parametrizations, or the continuity of the parameter space distribution. Since the three best parametrizations are similar for muon events, these events should have small $TD$ values. The $TD$
Figure 5–4: The $AP/TD$ distribution for different categories of events in run 47511: The colours represent the number of events per bin. See text for details.

parameter is essentially insensitive to noise, as muon events with significant amounts of noise have the same $TD$ values on average than similar muon events without noise. Consequently, muon events with significant amounts of noise have been identified using the $TD$ parameter.

5.3 Muon identification using the $AP/TD$ parameter space

Figure 5–4 shows 2D histograms of the $AP/TD$ distribution for different categories of events in run 47511. The colour scale represents the number of events per bin. The upper-left histogram shows the $AP/TD$ distribution for events with $Npix \geq 10$ in the run. This distribution possesses a characteristic shape, and covers
a broad range of $AP$ and $TD$ values. The upper-right histogram shows all of the events that were visually categorized as muon events as described in section 5.1. As expected, the muon events have larger $AP$ and smaller $TD$ values on average compared with all of the events with $Npix \geq 10$ in the run. The cluster of events with low $AP$ and high $TD$ values were incorrectly parametrized due to ring centers located outside the field of view of the camera. The lower-left histogram shows events that were labeled non-muon events. These events have smaller $AP$ values on average than the muon events as well as a broad distribution of $TD$ values. The lower-right histogram shows events that were labeled ambiguous. Many of the muon events occupy a unique region of the $AP/TD$ parameter space, where non-muon and ambiguous events are absent. This observation motivated the use of cuts on $AP$ and $TD$ together, by showing that a pure muon sample could be obtained by doing so. The details of these cuts will be described in section 5.5.

5.4 The stability of the $AP/TD$ parameter space

Figure 5–5 shows 2D histograms of the $AP/TD$ parameter space for a number of different runs. The upper-left histogram shows the distribution of $AP$ and $TD$ values for the events with $Npix \geq 10$ in run 40839, a 10 minute single-telescope run taken with T3 in May 2008. The shape of the distribution is similar to that of run 47511 shown in figure 5–5 and the features of the two histograms occur at the same location. Since run 40839 predates run 47511 by a year and a half, this indicates that the $AP/TD$ parameter space is stable over long periods of time. The upper-right histogram shows the events with $Npix \geq 10$ in run 40840, a 20 minute single-telescope run taken on T3 with an acrylic filter placed in front of the camera.
Figure 5–5: The $AP/TD$ distribution for different runs. The colours represent the number of events per bin. See text for details.
The acrylic filter caused a significant reduction in the light levels, and therefore the event rate. Despite this fact, the $AP/TA$ distribution of this run is similar to the distributions of the previously considered runs. This indicates that the shape of the $AP/TA$ parameter space is insensitive to varying light levels. The lower-left histogram shows the $AP/TA$ distribution for all events with $Npix \geq 10$ in run 40841, a run similar to 40839. This run was included for completeness and shows that the shape of the $AP/TA$ parameter space distribution does not fluctuate significantly on a local time scale. The lower-right histogram shows events with $Npix \geq 10$ from run 47511, the same run shown in figure 5-5. In this case, the L1 trigger pattern (as discussed in section 2.4) was used instead of the pixel intensity pattern for the analysis; just one vote per hit pixel was added to the appropriate bins of the accumulator array. The overall distribution is similar to the other distributions, indicating that the shape of the $AP/TA$ parameter space does not change significantly if binary data is used instead of data with pixel intensity values. This fact will be important when implementing the Hough Transform with hardware, since only the L1 trigger pattern will be available for use.

Changing the number and type of possible circle parametrizations used to implement the transform was found to change the locations of the features in the $AP/TA$ parameter space more than the previously discussed factors. In particular, when only circle parametrizations that are contained within the camera are used (see section 6.2), the overall shape of the parameter space remains the same, but the $AP$ values shift downwards and the $TD$ distribution becomes less broad. Because the $AP/TA$ parameter space was found to be mostly sensitive to changes involving the
circles used to implement the lookup table, cuts on $AP$ and $TD$ for the purpose of muon identification need not vary between data sets if the possible circles used to implement the Hough transform remain fixed.

5.5 Specific cuts on $N_{pix}$, $AP$, and $TD$

A location in the $AP/TD$ parameter space populated solely by muon events (see Figure 5–4) can be described by the region above a diagonal line, bounded by a vertical (constant $TD$) line. Cuts on $N_{pix}$ can also be used to help improve the efficiency of the cuts on $AP$ and $TD$. Specifically, if $A$, $B$, $C$, $D$ and $E$ represent free parameters to be determined, a region occupied only by muons can be defined:

$$AP > A \times TD + B$$
$$TD < C$$
$$D \leq N_{pix} \leq E$$

Parameters $A$, $B$, $C$, $D$ and $E$ were iterated manually so that only events categorized as muons or ambiguous were left from the sample of visually categorized events in run 47511 between event numbers 0 and 40841. The cuts resulting in the greatest number of passed events with this criterion were found to be:

$$AP > 0.011 \times TD + 6.6$$
$$TD < 182$$
$$10 \leq N_{pix} \leq 79$$
Figure 5–6: The $AP/TD$ distribution for $10 \leq Npix \leq 79$ of run 47511 for different categories of events. The red lines represent the cuts on $AP$ and $TD$.

Figure 5–6 shows the $AP/TD$ parameter space for all events in run 47511 with $10 \leq Npix \leq 79$, as well as the categorized events that pass this pixel cut. The region bounded by the red lines and the edge of the histograms represents the region of the $AP/TD$ parameter space defined by the cuts. Table 5–1 shows the distribution of categorized events between event numbers 0 to 40841 before and after the cuts. As measured on the sample of visually identified muons, the muon detection efficiency of our procedure in this range is approximately: $\frac{225}{795} \sim 0.28$. In order to verify the effectiveness of these cuts on an independent sample of events, the cuts were applied to the visually categorized events in run 47511 between event numbers 40842
Table 5–1: The distribution of visually categorized events before and after cuts from events 0 to 40842 in run 47511 using pixel intensity data.

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Muons</th>
<th>Non-muons</th>
<th>Ambiguous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before cuts</td>
<td>10921</td>
<td>795</td>
<td>7918</td>
<td>2208</td>
</tr>
<tr>
<td>After cuts</td>
<td>239</td>
<td>225</td>
<td>0</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 5–2: The distribution of visually categorized events before and after cuts from events 40842 to 81682 in run 47511 using pixel intensity data.

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Muons</th>
<th>Non-muons</th>
<th>Ambiguous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before cuts</td>
<td>11853</td>
<td>721</td>
<td>9109</td>
<td>2023</td>
</tr>
<tr>
<td>After cuts</td>
<td>228</td>
<td>210</td>
<td>0</td>
<td>18</td>
</tr>
</tbody>
</table>

and 81682. The results of this analysis are shown in table 5–2. The cuts produced a sample of events categorized as muons or ambiguous, with no non-muon events passing. The efficiency of the cuts in this range was found to be approximately: \( \frac{210}{721} \sim 0.29 \), indicating that the cuts effectively identify muons with a similar efficiency on an independent set of events.

The total number of events that passed the cuts in run 47511 was found to be 1617. These events were all verified by eye, and found to be mostly muon events or ambiguous. Five non-muon events passed the cuts. Figure 5–7 shows some of the events that passed. Motivated by the similarity of the \( AP/TD \) parameter space between runs as shown in Figure 5–5, the same cuts were applied to runs 40839, 40840 and 40841 using pixel intensity data. The results of this analysis are presented in table 5–3. It is clear that the selection criteria identify a highly pure sample of muon events in each of the runs.
Figure 5–7: A sample of muons with pixel intensity data passing cuts on $N_{pix}$, $AP$ and $TD$.

<table>
<thead>
<tr>
<th>Run</th>
<th>Events</th>
<th>Events passing cuts</th>
<th>Fraction</th>
<th>False positives</th>
</tr>
</thead>
<tbody>
<tr>
<td>47511</td>
<td>274991</td>
<td>1617</td>
<td>0.0059 ± 0.0001</td>
<td>5</td>
</tr>
<tr>
<td>40839</td>
<td>184048</td>
<td>1105</td>
<td>0.0060 ± 0.0002</td>
<td>2</td>
</tr>
<tr>
<td>40840</td>
<td>166224</td>
<td>730</td>
<td>0.0044 ± 0.0002</td>
<td>4</td>
</tr>
<tr>
<td>40841</td>
<td>184451</td>
<td>1101</td>
<td>0.0060 ± 0.0002</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5–3: The number of events scanned, number passing cuts, fraction passing cuts and false positives for four runs using pixel intensity data.
<table>
<thead>
<tr>
<th>Run</th>
<th>Events</th>
<th>Events passing cuts</th>
<th>Fraction</th>
<th>False positives</th>
</tr>
</thead>
<tbody>
<tr>
<td>47511</td>
<td>275046</td>
<td>1594</td>
<td>0.0058 ± 0.0001</td>
<td>0</td>
</tr>
<tr>
<td>40839</td>
<td>184053</td>
<td>1057</td>
<td>0.0057 ± 0.0002</td>
<td>0</td>
</tr>
<tr>
<td>40840</td>
<td>166226</td>
<td>804</td>
<td>0.0048 ± 0.0002</td>
<td>0</td>
</tr>
<tr>
<td>40841</td>
<td>184453</td>
<td>1003</td>
<td>0.0054 ± 0.0002</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5–4: The number of events scanned, number passing cuts, fraction passing cuts and false positives for four runs using binary pixel data.

Motivated by the similar shape of the $AP/TD$ parameter space for binary data as shown in figure 5–5, the same cuts were applied to binary data from run 47511. In the range over which events were visually categorized (event numbers 1-81682), 475 events passed the cuts. Assuming the same ratio of ambiguous events passing cuts to the total number of events passing cuts found with pixel intensity data, the efficiency of the cuts using binary data in this range was found to be approximately: $\frac{475}{1516} \times (1 - \frac{32}{467}) \sim 0.29$. The number of events that passed the cuts in run 47511 using binary data was found to be 1594. All of these events were visually inspected, and were found to consist entirely of muons or ambiguous events. In particular, no non-muon events were found to pass the cuts. Figure 5–8 shows a sample of these events.

The same cuts were also applied to runs 40839, 40840 and 40841 using binary pixel data. The results of this analysis are shown in table 5–4. These results indicate that the cuts are able to produce a pure muon sample using binary data with efficiencies similar to those found when using the same cuts on data with pixel intensity values. Since these cuts have been demonstrated to work with binary data, which could be used to identify muons using a hardware trigger system, the rest of the analysis in this thesis will pertain only to binary data.
Figure 5–8: A sample of binary data muons passing cuts on $N_{pix}$, $AP$ and $TD$. 
6.1 Selection of complete muon rings

In order to use muon rings for calibration, the sample of events found by using cuts on \( N_{\text{pix}} \), \( AP \) and \( TD \) must be filtered so that only events that are useful for calibration (as described in section 3.4) remain. These events consist of contained and azimuthally complete muon rings, which will be described in detail below.

6.2 Contained muon rings

Contained muon rings have light distributions that are fully contained within the field of view of the camera. In other words, the rings are not truncated by the edge of the camera. Contained muon rings allow the measurement of the signal produced by light from all azimuthal angles, which is needed for relating the amount of light incident on the reflector to the size of the muon ring, as described in section 3.4. Figure 6–1 shows examples of contained muon rings, and figure 6–2 shows examples of muon rings that are not contained. The red circles correspond to the best parametrizations of the muon rings, \((x_1, y_1, r_1)\). If \( D_{\mu c} \) is the distance from the center of the muon ring to the center of the camera (the angle of incidence), \( R_{\mu} \) is the radius of the muon ring (the Cherenkov angle) and \( R_c \) is the radius of the camera (half the angular field of view), then the containedness condition is:

\[
D_{\mu c} + R_{\mu} < R_c.
\]
Figure 6–1: Examples of contained muon rings.

Figure 6–2: Examples of non-contained muon rings.
In this work, $R_\mu$ is assumed to be the radius of the best parametrization, $r_1$, and $D_{\mu c}$ is calculated using the distance from the center of the best parametrization, $(x_1, y_1)$, to the center of the camera. The radius of the camera, $R_c$ is taken to be a free parameter, as the edge of the camera is not perfectly circular. The specific value of $R_c$ used in this work, and the results of implementing the containedness condition using this value will be discussed in section 6.4.

Another approach to the selection of contained muon rings is to implement the containedness condition at the level of the lookup table. This means only using circles that pass the containedness condition for generating the lookup table used to implement the Hough transform (described in section 4.4). This reduces the number of circles used to implement the lookup table from 12475 to 4407 and shifts the scale and location of features in the $AP/TD$ parameter space, but leaves the shape of the distribution intact. This approach is the most computationally efficient method for selecting contained rings, but does not offer a means to estimate detection efficiencies as described in section 5.5.

6.3 Azimuthally complete muon rings

Azimuthally complete muon rings consist of rings that form complete circles (as opposed to arcs) in the camera plane. Azimuthally complete rings are caused by muons that have impact parameters that are less than the radius of the reflector, as described in section 3.2. Azimuthally complete muon rings allow the best estimates to be made of the number of photons incident on the reflector, making them the best events for calibration purposes as described in section 3.4. Examples of contained, azimuthally complete muon rings are shown in figure 6–3 and examples of contained,
Figure 6–3: Examples of contained, azimuthally complete muon rings.

Figure 6–4: Examples of contained, azimuthally incomplete muon rings.
azimuthally incomplete muon rings are shown in figure 6–4. Selecting azimuthally complete rings was accomplished by using two parameters: \(NP_R\) and \(C_{Dist}\).

The \(NP_R\) (number of pixels in the ring) parameter consists of the number of pixels within a distance of one PMT diameter from any of the three best parametrizations. The blue pixels in Figures 6–3 and 6–4 correspond to the pixels included in the \(NP_R\) count. Azimuthally complete muon rings of a given radius tend to have larger \(NP_R\) values than azimuthally incomplete rings. The \(NP_R\) parameter must be normalized in order to account for the effects of different ring radii. Specifically, rings with large radii tend to have larger \(NP_R\) values than rings with small radii for the same amount of azimuthal completeness. The normalization is accomplished by dividing \(NP_R\) by the radius of the best parametrization of the event, defining the new variable \(NP_{R1}\). Azimuthally complete muon rings tend to have larger \(NP_{R1}\) values than azimuthally incomplete muon rings.

The \(C_{Dist}\) (centroid distance) parameter consists of the distance from the center of the best parametrization, \((x_1, y_1)\) (the red dot in figures 6–3 and 6–4) to the centroid of the pixels in the ring, \((x_c, y_c)\) (the small blue dot in figures 6–3 and 6–4). Specifically:

\[
C_{Dist} = \sqrt{(x_1 - x_c)^2 + (y_1 - y_c)^2}
\]

where \((x_c, y_c) = \left( \sum_{i=1}^{NP_R} \frac{x_i}{NP_R}, \sum_{i=1}^{NP_R} \frac{y_i}{NP_R} \right)\) and \((x_i, y_i)\) represents the coordinates of the center of the \(i^{th}\) pixel included in the \(NP_R\) count. Muon rings with circular symmetry tend to have ring centroids near the center of the ring, whereas muon rings without circular symmetry have ring centroids that are far from the center.
of the ring. Therefore, azimuthally complete muon rings of a given radius tend to have smaller $C_{Dist}$ values than azimuthally incomplete rings. The $C_{Dist}$ parameter must also be normalized in order to correct for the effects of different ring radii. Specifically, muon rings with small radii have smaller $C_{Dist}$ values than muon rings with large radii for a given amount of azimuthal completeness. The normalization is accomplished by dividing $C_{Dist}$ by the radius of the best parametrization of the event $r_1$, defining the new variable $\frac{C_{Dist}}{r_1}$. Azimuthally complete muon rings tend to have smaller $\frac{C_{Dist}}{r_1}$ values than azimuthally incomplete rings.

The observation that azimuthally complete muon rings tend to have high $\frac{NPR}{r_1}$ values and small $\frac{C_{Dist}}{r_1}$ values motivated the creation of a new parameter:

$$\frac{C_{Dist}}{NPR} = \frac{C_{Dist}}{NPR} \equiv C/N$$

The $C/N$ parameter combines the two previously mentioned ideas for determining azimuthal completeness, and eliminates the need to normalize by the radius of the best parametrization. Azimuthally complete muon rings tend to have lower $C/N$ values than azimuthally incomplete rings. The details of the specific cuts on $C/N$ used and the corresponding results will be described in section 6.4

### 6.4 Specific containedness and azimuthal completeness cuts and results

In this work, muons from run 47511 that passed the cuts described in section 5.5 were filtered by the containedness condition described in section 6.2. $R_c$ was taken to be 11 PMT diameters, corresponding to approximately 340 mm or 1.65 degrees. This value corresponds to the distance from the middle of the center pixel of the
camera to the middle of the pixel at the outer edge of the central horizontal row of the camera. Filtering the events from run 47511 that passed the cuts described in section 5.5 by this condition reduced the number of selected events from 1594 to 365. Figure 6–1 shows events that passed this containedness condition, and figure 6–2 shows events that did not pass.

The contained muons were then filtered using various cuts on $C/N$. The muons passing each of these cuts were visually inspected in order to determine which cut produced a sample of azimuthally complete muons. The cuts took the form of $C/N < D$ where $D$ was a free parameter. Various values of $D$ were tried, covering the range from 0.5 mm to 2.5 mm in increments of 0.25 mm. Table 6–1 shows the number of events found for different values of $D$. The value of $D$ that resulted in a sample of mostly azimuthally complete (at least $\sim \frac{3}{4}$ complete) muon rings was 2.0 mm. Filtering the muons from run 47511 that passed the cuts described in section 5.5 by this condition and the containedness condition reduced the number of events from 1594 to 178. This corresponds to a fraction of $\frac{178}{1594} = 0.112$ of the original sample. The muon rings shown in figure 6–3 correspond to contained events passing the azimuthal completeness condition and the muon rings in figure 6–4 correspond to contained events that did not pass.

In order to determine the reliability of the containedness and azimuthal completeness cuts on other runs, the same cuts were applied to binary data muons passing the cuts described in section 5.5 from runs 40839, 40840 and 40841. The results are presented in table 6–2. The fraction of the original sample remaining after the containedness and azimuthal completeness cuts for runs 40839 and 40841 is comparable
<table>
<thead>
<tr>
<th>$D \ (mm)$ ($C/N \ &lt;$)</th>
<th>Events found</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>204</td>
</tr>
<tr>
<td>2.25</td>
<td>191</td>
</tr>
<tr>
<td>2.0</td>
<td>178</td>
</tr>
<tr>
<td>1.75</td>
<td>162</td>
</tr>
<tr>
<td>1.5</td>
<td>145</td>
</tr>
<tr>
<td>1.25</td>
<td>124</td>
</tr>
<tr>
<td>1.0</td>
<td>101</td>
</tr>
<tr>
<td>0.75</td>
<td>67</td>
</tr>
<tr>
<td>0.5</td>
<td>37</td>
</tr>
</tbody>
</table>

Table 6–1: Number of events found for various values of $D$.

to run 47511, indicating that these cuts are stable for typical data runs. However, the fraction of events remaining from the original sample of muons from run 40840 (described in section 5.4) after containedness and azimuthal completeness cuts is significantly lower than the other runs. This indicates that these cuts are sensitive to varying light levels. This is due to the use of the $NPR$ parameter for determining azimuthal completeness, which is smaller for runs with reduced light levels, since fewer pixels pass the L1 trigger condition. However, since normal data will not be taken with reduced light levels, the $C/N$ azimuthal completeness cut can still be used to select muons for calibration under normal circumstances. The events that passed the cuts in each of the processed runs were visually inspected and found to consist of contained and mostly azimuthally complete (at least $\sim \frac{3}{4}$ complete) muon rings. This demonstrates that these cuts can be used on normal data runs to identify muons useful for calibration.
<table>
<thead>
<tr>
<th>Run</th>
<th>Events in the original sample</th>
<th>Events passing cuts</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>47511</td>
<td>1594</td>
<td>178</td>
<td>$0.112 \pm 0.008$</td>
</tr>
<tr>
<td>40839</td>
<td>1057</td>
<td>137</td>
<td>$0.13 \pm 0.01$</td>
</tr>
<tr>
<td>40840</td>
<td>804</td>
<td>8</td>
<td>$0.010 \pm 0.004$</td>
</tr>
<tr>
<td>40841</td>
<td>1003</td>
<td>121</td>
<td>$0.12 \pm 0.01$</td>
</tr>
</tbody>
</table>

Table 6–2: Number of events found using the cuts described in section 5.5 (the original sample), the number of events from the original sample passing containedness and azimuthal completeness cuts, and the fraction from the original sample remaining. Run 40840 was taken with reduced light levels, causing a smaller fraction of events to pass the cuts due to the use of the NPR variable.
CHAPTER 7
Comparison of the Hough transform-based selection technique with the standard VERITAS muon selection technique

7.1 Details of the standard VERITAS muon selection technique

The standard VERITAS muon selection technique, implemented in the VEGAS and Event Display offline analysis packages, consists of two parts: the parametrization algorithm and the muon identification criteria. The parametrization algorithm calculates the distance from a location in the camera, (starting from the center) to each of the hit pixels in the image. The average of the distances is taken to be the radius of the ring and the standard deviation of the distances is taken to be the uncertainty on the radius. The location of the center of the assumed ring is iterated until the standard deviation of the distance measurements is minimized. The location \((x, y)\) of the center of the assumed ring that gives the lowest value of the standard deviation of the distances is taken to be the best location of the center of the assumed ring. The average of the distances from that location to the hit pixels in the event, \(r\), is taken to be the radius of the assumed ring. The best parametrization of the event \((x, y, r)\) is used to identify muons as follows: In order to be considered a muon event, 70 percent of the hit pixels must be within a distance of one pixel diameter from the best parametrization. In addition, there must be at least two hit pixels in each octant of the best parametrization within a distance of one PMT diameter from the circle. Additional cuts are used, which are described in Appendix A.
7.2 A comparison of both techniques

The standard VERITAS muon selection technique described in section 7.1 was compared with the Hough transform-based technique described in chapters 4, 5 and 6. This was accomplished by comparing lists of event numbers found by the two techniques on the same run, 47511. One list consisted of the event numbers of the 178 events in run 47511 passing our Hough transform-based muon selection criteria. The other list consisted of the event numbers of the 106 events in run 47511 passing the standard VERITAS muon selection criteria. The results of this analysis are described below.

The 93 events that were found by both techniques consist of contained and mostly azimuthally complete (at least $\sim \frac{5}{6}$ complete) muon rings. These events also have a high degree of azimuthal symmetry, low levels of scatter (variations in the ring radius) and few non-ring pixels on average. Figure 7–1 shows examples of events found by both techniques.

The 85 events found only by the Hough transform-based technique consist of contained and mostly azimuthally complete (at least $\sim \frac{3}{4}$ complete) muon rings. These events have more non-ring hit pixels on average than the events found only by the standard VERITAS muon selection technique and by both techniques. These events also have less azimuthal symmetry and more scatter than the events found by both techniques. Figure 7–2 shows examples of events found only by the Hough transform-based technique.

The 13 events found only by the standard VERITAS muon selection technique consist of contained and mostly azimuthally complete (at least $\sim \frac{5}{6}$ complete) muon
rings. These events are less azimuthally symmetric and have more scatter on average than events found by both techniques. These events were investigated to determine why they did not pass the Hough transform-based muon selection criteria. Eight events (rings with small radii) failed the cuts on $AP$, one event failed the $C/N$ cut, one event failed both the $AP$ and $C/N$ cuts, and one event failed the contained-ness condition. Figure 7–3 shows examples of events found only by the standard VERITAS muon selection technique.

Clearly, the Hough transform-based technique finds more muon events that the standard VERITAS muon selection technique, when using the cuts described earlier. The Hough transform-based technique also finds events that have more non-ring hit pixels on average, indicating that it is less sensitive to noise than the standard VERITAS muon selection technique. Rings with smaller radii were observed to have smaller $AP$ values on average, and therefore pass cuts on $AP$ less often than rings with larger radii. These events make up the majority of the events found only by the standard VERITAS muon selection technique. This suggests modifying the $AP$ parameter to compensate for this effect.
Figure 7–1: Examples of events found by both techniques.

Figure 7–2: Examples of events found only by the Hough transform-based muon selection technique.
Figure 7–3: Examples of events found only by the standard VERITAS muon selection technique. The variables coloured red did not pass the Hough transform-based muon selection criteria.
8.1 Summary

A circular Hough transform has been implemented with a lookup table and applied to VERITAS data for the purpose of identifying muons. In order to test the effectiveness of parameters derived from the accumulator array for muon identification, 22774 events with pixel intensity values were categorized by eye as muons, non-muons or ambiguous. The categorization process allowed the identification of three variables for muon identification: \( AP \), \( TD \) and \( Npix \). In addition, it motivated the general form of the cuts on these quantities. The cuts on \( AP \), \( TD \) and \( Npix \) were optimized using half of the visually categorized events in run 47511, yielding a pure muon sample with a detection efficiency of approximately 28 percent. These cuts were then applied to the other half of the visually categorized events, producing a pure muon sample with an efficiency of approximately 29 percent. This indicates that the cuts produce a pure muon sample with a similar efficiency on an independent set of events. The cuts were then applied to all of the events in four separate runs, 47511, 40839, 40840 and 40841, yielding a highly pure muon sample in each case. The normal runs were found to have similar muon detection rates, indicating that the cuts on \( Npix \), \( AP \) and \( TD \) are stable for these runs. The run with the acrylic filter placed in front of the camera, run 40840, showed a 25 percent reduction in the detection rate. The cuts on \( Npix \), \( AP \) and \( TD \) were applied to binary data, and
were found to produce a pure muon sample with efficiencies similar to those found using pixel intensity data. Since this type of data could be used to implement the Hough transform-based muon selection technique in a hardware trigger, binary data was used for the rest of the analysis.

In order to select contained and azimuthally complete muon rings for calibration purposes, additional cuts were used. The selection of contained rings was accomplished by using the best parametrization of the event, which was found to be effective. The selection of azimuthally complete muon rings was accomplished by the use of the $C/N$ parameter, which combined two ideas for determining azimuthal completeness into one variable. Cuts on $C/N$ were found to produce a mostly azimuthally complete (at least $\sim \frac{3}{4}$ complete) sample of muon rings from samples of events passing cuts on $N_{pix}$, $AP$ and $TD$ and the containedness condition. Containedness cuts and cuts on the $C/N$ parameter were found to select contained and azimuthally complete muon rings in normal data with a consistent efficiency. However, the azimuthal completeness cut was found to be inefficient at selecting events from run 40840, the run with the acrylic filter placed in front of the camera. This was due to the use of the $NPR$ variable in the calculation of the $C/N$ parameter, as runs with reduced light levels have fewer triggered pixels and therefore smaller $NPR$ values. This suggests modifying the method used to determine azimuthal completeness.

The muons selected using the Hough transform-based technique were compared to the muons selected by the standard VERITAS muon selection technique. The Hough transform-based technique was found to select more muon events than the
standard VERITAS muon selection technique, using the cuts described in this thesis. The Hough transform-based technique also finds muons with more non-ring pixels on average, indicating that the technique is less sensitive to noise than the standard VERITAS muon selection technique. Muon rings with smaller radii were found to have lower $AP$ values, and thus pass cuts on $AP$ less often than rings with larger radii. This suggests modifying the $AP$ parameter to compensate for this effect.

### 8.2 Future work

The Hough transform-based muon selection technique may be improved in a number of ways. For example, different cuts on $N\text{pix}$, $AP$, $TD$ and $C/N$ may produce higher efficiencies than the ones described in this thesis. Different variables for muon identification may also improve performance. In particular, the modification of the $AP$ parameter to correct for the effect of ring radius is motivated by the results of comparing both muon identification techniques. Higher dimensional versions of the $TD$ variable may also increase efficiency. In addition, the $C/N$ parameter was found to be light dependent, which suggests using a different method for selecting azimuthally complete muon rings. Using a different set of parametrized circles to construct the lookup table may also improve efficiency. Specifically, using a greater number of $(x, y)$ locations as well as $(x, y)$ locations outside field of view of the camera.

Certain properties of the Hough transform-based muon selection technique may be investigated. Specifically, the speed performance of the technique may be investigated in order to determine its suitability for use in a hardware trigger system. In
addition, the noise sensitivities of the Hough transform-based technique and the standard VERITAS muon selection technique may be characterized in order to improve our understanding of how both techniques compare to each other.

The Hough transform-based muon selection technique will eventually be implemented in the VERITAS offline analysis packages, VEGAS and Event Display. This will allow a complementary muon selection algorithm to be used for muon calibration work already in progress. In addition, the technique may be implemented in dedicated muon triggers on each telescope, allowing the continuous collection of muon calibration data during normal operations.
APPENDIX A
The standard VERITAS muon selection technique: additional cuts

In addition to the primary muon selection criteria, described in section 7.1, the following additional cuts were used:

• The event must have a non-zero size in digital counts (DC).

• The radius of the best parametrization of the event must be greater than 0.5 degrees.

• The standard deviation of the distances from the hit pixels to the location of center of the best parametrization must be less than 0.1 degrees.

• The summed charge of the pixels within one pixel diameter (0.15 degrees) of either side of the best parametrization of the event (muon size) must be less than 10000 DC.

• The muon size (described above) divided by a muon impact parameter correction factor (described in [6] and [15]) must be less than 10000 DC.
• The distance from the center of the camera to the center of the best parametrization of the event plus the radius of the ring must be less than 1.55 degrees.

• The distance from the center of the camera to the center of the best parametrization must be less than 0.5 degrees.
References


