The Virasoro fusion kernel and its applications

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November 26th, 2018
Based on arXiv: 1811.05710 [hep-th]

Quantum Regge Trajectories and the Virasoro Analytic Bootstrap

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The lightcone bootstrap

The fusion kernel

Kernel and CFT data

Large c limits
Outline

1. The lightcone bootstrap
2. The fusion kernel
3. Kernel and CFT data
4. Large $c$ limits
All CFTs have OPE (here scalar)

\[ \phi(x)\phi(0) = \sum_{O} f_{\phi\phi\phi} C(x, \partial) O(0) \]
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\]

Consider using it for 12 and 34 (s-channel) in \(d \geq 3\)

\[
\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle = \sum_\mathcal{O} f_{\phi\phi\mathcal{O}}^2 G_{\Delta\mathcal{O},\ell\mathcal{O}}^{\Delta\phi}(z, \bar{z}) \frac{(x_{12})^{2\Delta\phi}(x_{34})^{2\Delta\phi}}{}
\]

with \(G_{\Delta\mathcal{O},\ell\mathcal{O}}^{\Delta\phi}\) conformal blocks and \(z, \bar{z}\) conformal cross-ratios
Conformal block decomposition

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with $G_{\Delta\mathcal{O},\ell\mathcal{O}}^{\Delta\phi}$ conformal blocks and $z, \bar{z}$ conformal cross-ratios

Write sum in terms of twist $\tau = \Delta - \ell$
Crossing symmetry

Can do 14 and 23 instead (t-channel) and get same thing

\[ \sum_{O} f_{\phi \phi O}^2 G_{\tau, \ell}^{\Delta \phi} (z, \bar{z}) = \]

\[ \left( \frac{z \bar{z}}{(1 - z)(1 - \bar{z})} \right)^{\Delta \phi} \sum_{O'} f_{\phi \phi O'}^2 G_{\tau', \ell'}^{\Delta \phi} (1 - z, 1 - \bar{z}) \]
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Take $\bar{z} \to 1$, t-channel blocks behave as

$$G_{\Delta', \ell'}^\phi(1 - z, 1 - \bar{z}) \approx (1 - \bar{z})^{\frac{\tau'}{2}} K_{\Delta' + \ell'}(1 - z)$$

$\Rightarrow$ t-channel dominated by identity!
Lightcone limit

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Further take $z \to 0$, s-channel blocks behave as

$$G_{\tau \ell}^{\Delta \phi}(z, \bar{z}) \approx z^{\frac{\tau}{2}} \log (1 - \bar{z})$$
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Crossing symmetry becomes

$$\sum_{\tau, \ell} f_{\phi\phi\phi}^2 z^{\frac{\tau}{2}} \log (1 - \bar{z}) = \frac{z^{\Delta \phi}}{(1 - \bar{z})^{\Delta \phi}} + \ldots$$
Double twist operators

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Explicitly inverting crossing gives the OPE coefficients
Mean Field Theory

t-channel identity $\Rightarrow$ s-channel "double twists"
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Reproduces Mean Field Theory: CFT with correlators given by Wick contractions, contain only double twist operators
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RESULT: Every CFT behaves as MFT at large spin
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RESULT: Every CFT behaves as MFT at large spin

Including subleading operators in t-channel gives corrections to OPE and anomalous dimensions

$$\gamma_{n,\ell} \sim \frac{1}{\ell^\tau}$$
Regge trajectories
Inversion formula

Can write 4-point function as

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle \sim \sum_{\ell=0}^{\infty} \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} d\Delta C(\Delta,\ell)G_{\Delta,\ell}(z,\bar{z})$$

where $C$ has poles at physical operator with residues giving the OPE coefficients
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Simon’s formula inverts this

$$C(\Delta, \ell) \propto \int_{0}^{1} \int_{0}^{1} dzd\bar{z} M_{\Delta,\ell}(z, \bar{z}) \text{dDisc}[\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle]$$
6j symbols

Inserting identity in inversion formula gives MFT result
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$\Rightarrow$ 6j symbols rewrite t-channel data into s-channel data
What is wrong in 2d?
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We will take finite $c$ and reproduce their results.
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Conformal transformations factorize into holomorphic and anti-holomorphic
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⇒ Conformal blocks factorize

\[ G(z, \bar{z}) = \mathcal{F}(h|z)\bar{\mathcal{F}}(\bar{h}|\bar{z}) \]

with \( h = \Delta + \ell \) and \( \bar{h} = \Delta - \ell \)
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Crossing symmetry for \( \langle O_1(0)O_2(z, \bar{z})O_2(1)O_1(\infty) \rangle \) is now

\[
\sum_s (f_{12s})^2 \mathcal{F}_S(h_s, z)\bar{\mathcal{F}}_S(\bar{h}_s, \bar{z}) = \\
\sum_t f_{11t} f_{22t} \mathcal{F}_T(h_t, 1-z)\bar{\mathcal{F}}_T(\bar{h}_t, 1-\bar{z})
\]
Liouville notation

Need to use new notation:

\[ c = 1 + 6Q^2 \quad , \quad Q = b + b^{-1} \quad , \quad h = \alpha(Q - \alpha) \]

\[(h, c) \Rightarrow (\alpha, b)\]
Liouville notation

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\[(h, c) \Rightarrow (\alpha, b)\]

Operators separate in two ranges:

Discrete: \(0 < h < \frac{c-1}{24} \iff 0 < \alpha < \frac{Q}{2}\)

Continuum: \(h \geq \frac{c-1}{24} \iff \alpha = \frac{Q}{2} + iP\)
Definition of the kernel

Rewrite t-channel (holomorphic) Virasoro blocks into s-channel blocks

\[ \mathcal{F}_T(\alpha_t, 1 - z) = \int_C \frac{d\alpha_s}{2i} S_{\alpha_s \alpha_t} \mathcal{F}_S(\alpha_s, z) \]
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Poles at \( \alpha_s = \alpha_1 + \alpha_2 + mb + nb^{-1} \) and reflexions \( \alpha \to Q - \alpha \)

- For \( \alpha_t = 0 \), single poles
- For \( \alpha_t \neq 0 \), double poles
When $\alpha_1 + \alpha_2 > \frac{Q}{2}$, $C$ is simple
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Analytic structure

When $\alpha_1 + \alpha_2 < \frac{Q}{2}$, poles at $\alpha_m = \alpha_1 + \alpha_2 + mb$ can cross axis.
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Support of the kernel

- For $\alpha_1 + \alpha_2 > \frac{Q}{2}$,

$$\mathcal{F}_T(\alpha_t) = \int_0^\infty dP \, S_{\alpha_s \alpha_t} \mathcal{F}_S \left( \alpha_s = \frac{Q}{2} + iP \right)$$
Support of the kernel

- For $\alpha_1 + \alpha_2 > \frac{Q}{2}$,

$$\mathcal{F}_T(\alpha_t) = \int_0^\infty dP \mathbb{S}_{\alpha_s \alpha_t} \mathcal{F}_S \left( \alpha_s = \frac{Q}{2} + iP \right)$$

- For $\alpha_1 + \alpha_2 < \frac{Q}{2}$,

$$\mathcal{F}_T(\alpha_t) = -2\pi \sum_m \text{Res}_{\alpha_s = \alpha_m} \left\{ \mathbb{S}_{\alpha_s \alpha_t} \mathcal{F}_S(\alpha_s) \right\}$$

$$+ \int_0^\infty dP \mathbb{S}_{\alpha_s \alpha_t} \mathcal{F}_S \left( \alpha_s = \frac{Q}{2} + iP \right)$$

with sum over $\alpha_m < \frac{Q}{2}$
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Rewrite t-channel into s-channel with kernel tells us what must be there in the s-channel to reproduce what appears in t-channel.
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Consider $\alpha_1 + \alpha_2 < \frac{Q}{2}$ and $\bar{\alpha}_1 + \bar{\alpha}_2 > \frac{Q}{2}$ and individual t-channel exchange

$$\int d\alpha_s d\bar{\alpha}_s \rho_{12s} F_S(\alpha_s) \bar{F}_S(\bar{\alpha}_s) = \int_0^\infty d\bar{P} \bar{S}_{\alpha_s \alpha_t} F_S \left( \bar{\alpha}_s = \frac{Q}{2} + i\bar{P} \right)$$

$$f_{11t} f_{22t} \left[ -2\pi \sum_m \text{Res} \left\{ S_{\alpha_s \alpha_t} F_S(\alpha_s) \right\} + \int_0^\infty dP S_{\alpha_s \alpha_t} F_S \left( \alpha_s = \frac{Q}{2} + iP \right) \right]$$
Virasoro MFT

What is needed to reproduce identity $\alpha_t = \bar{\alpha}_t = 0$?
Virasoro MFT

What is needed to reproduce identity $\alpha_t = \bar{\alpha}_t = 0$?

1. Family of operators with $\alpha = \alpha_m < \frac{Q}{2}$ (in discrete spectrum) for each $\bar{\alpha}$ in continuum ⇒ “Quantum” Regge trajectories

2. Operators with $\alpha$ and $\bar{\alpha}$ in continuum
What is needed to reproduce identity $\alpha_t = \bar{\alpha}_t = 0$?

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OPE coefficients of Regge operators given by

$$\rho_{12m} = -2\pi \bar{S}_{\bar{\alpha}I} \text{Res}_{\alpha_s = \alpha_m} S_{\alpha_s I}$$

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This is called Virasoro Mean Field Theory!
Assume other operators give small corrections

$$(\rho_{12m} + \delta \rho_{12m}) \mathcal{F}_S(\alpha_m + \delta \alpha_m) \tilde{\mathcal{F}}_S \approx \tilde{\mathcal{F}}_S (\rho_{12m} \mathcal{F}_S(\alpha_m) + \delta \rho_{12m} \mathcal{F}_S(\alpha_m) + \rho_{12m} \delta \alpha_m \partial \mathcal{F}_S(\alpha_m))$$
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This leads to

\[\delta \alpha_m = f_{11t} f_{22t} \frac{\bar{S}_{\bar{\alpha}_m \bar{\alpha}_t}}{\bar{S}_{\bar{\alpha}_m}^{\bar{S}}} \frac{d\text{Res}}{\text{Res}} \frac{S_{\alpha_s \alpha_t}}{S_{\alpha_s}^{\bar{S}}}\]

\[\delta \rho_{12m} = -2\pi f_{11t} f_{22t} \bar{S}_{\bar{\alpha}_m \bar{\alpha}_t} \text{Res} \frac{S_{\alpha_s \alpha_t}}{S_{\alpha_s}^{\bar{S}}}\]

where dRes means the coefficient of double pole
Why dRes?

Taylor expanding double pole at $x = x_0$ gives

$$s(x)f(x) = \left( \frac{d\text{Res}(s)}{(x - x_0)^2} + \frac{\text{Res}(s)}{x - x_0} + s(x_0) \right)$$

$$\times \left( f(x_0) + (x - x_0)f'(x_0) \right)$$
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$$= \frac{f(x_0) \ d\text{Res}(s)}{(x - x_0)^2} + \frac{f(x_0) \ \text{Res}(s)}{x - x_0} + f'(x_0) \ \text{Res}(s) + \ldots$$
Why dRes?

Taylor expanding double pole at $x = x_0$ gives

$$s(x)f(x) = \left( \frac{d\text{Res}(s)}{(x - x_0)^2} + \frac{\text{Res}(s)}{x - x_0} + s(x_0) \right) \times \left( f(x_0) + (x - x_0)f'(x_0) \right)$$

$$= \frac{f(x_0)\text{dRes}(s)}{(x - x_0)^2} + \frac{f(x_0)\text{Res}(s)}{x - x_0} + f'(x_0)\text{Res}(s) + \ldots$$

$$\Rightarrow \text{Res}(sf) = f(x_0)\text{Res}(s) + f'(x_0)\text{dRes}(s)$$
At large $\bar{\alpha}_s$

$$\delta \alpha_m \sim e^{-2\pi \bar{\alpha}_t \sqrt{\ell_s}}$$
Large spin asymptotics

At large $\bar{\alpha}_s$

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$\Rightarrow$ identity dominates at large spin!
Large spin asymptotics

At large $\bar{\alpha}_s$

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⇒ identity dominates at large spin!

Spectrum of Quantum Regge trajectories at large spin:

$$h_m = h_1 + h_2 + m - 2(\alpha_1 + mb)(\alpha_2 + mb) + m(m+1)b^2 + \delta h_m$$
Quantum Regge trajectories
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Global limit

Reproduce global results with $c \to \infty$ and $h_i$ fixed
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Infinite number of trajectories with

\[ h_m = h_1 + h_2 + m + O(b^2) \]
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Checks:

1. Reproduce MFT from VMFT (exchange of identity)
2. Other t-channel reproduced
3. Next order in identity exchange gives \( T \)
Semiclassical limit

Again \( c \to \infty \) but some operators heavy \( h \sim c \)
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When $m \ll b^{-1} \sim \sqrt{c}$, $h_1 = O(c) < \frac{c}{24}$ and $h_2 = O(1)$, recover

$$h_m \approx h_1 + \sqrt{1 - \frac{24h_1}{c}}(h_2 + m)$$

same as FKW
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When $m \ll b^{-1} \sim \sqrt{c}$, $h_1 = O(c) < \frac{c}{24}$ and $h_2 = O(1)$, recover

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When further take $\frac{h_1}{c} \ll 1$, recover

$$h_m \approx h_1 + h_2 - \frac{12h_1h_2}{c}$$

which can be derived from inversion formula
Summary

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4. Corrections to trajectories
5. Large c limits
Many other applications

1. Virasoro blocks at late time (information paradox)
2. Gravity interpretation
3. $z \rightarrow 1$ limit of Virasoro blocks
4. HHLL Virasoro blocks
5. 2d lightcone bootstrap